Operational Semantics

Operational semantics provides a way of understanding what a program means by mimicking, at a high level, the operation of a computer executing the program. Operational semantics falls under two broad classes: *big-step* operational semantics, which specifies the entire operation of a given expression or statement; and *small-step* operational semantics, which specifies the operation of the program one step at a time. Both are powerful tools for verifying the correctness and other desired properties of programs.

Exercises

1. Use the big-step operational semantics rules for the WHILE language to write a well-formed derivation with $\langle E, y := 3; \texttt{if } y > 1 \texttt{ then } z := y \texttt{ else } z := 2\rangle \Downarrow E[y \mapsto 3; z \mapsto 3]$ as its conclusion. Make sure to indicate which rule you used to prove each premise or conclusion.

2. For homework 2, you will be partially proving that if a statement terminates, then the big- and small-step semantics for WHILE will obtain equivalent results; i.e.,

$$\forall S \in \texttt{Stmt}.\forall E, E' \in \texttt{Var} \mapsto \mathbb{Z}.\langle E, S\rangle \to^* \langle E', \texttt{skip}\rangle \iff \langle E, S\rangle \Downarrow E'$$

You will prove this by induction on the structure of derivations for each direction of $\iff$.

For your homework proof, you are only required to show

- The base case(s).
- The inductive case for `assign` and for `let` using the semantics developed in question 1 of the homework.

You may assume that this property holds for arithmetic and boolean expressions, i.e., you may assume the following hold:

$$\forall a \in \texttt{AExp}.\forall n \in \mathbb{Z}.\langle E, a\rangle \to_a^* n \iff \langle E, a\rangle \Downarrow_a n \tag{1}$$

$$\forall P \in \texttt{BExp}.\forall b \in \{\texttt{true}, \texttt{false}\}.\langle E, P\rangle \to_b^* b \iff \langle E, P\rangle \Downarrow_b b \tag{2}$$

You may also assume the small-step if congruence of $\langle E, S\rangle \to^* \langle E', S'\rangle$:

$$\frac{\langle E, P\rangle \to_b^* P'}{\langle E, \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2\rangle \to^* \langle E, \texttt{if } P' \texttt{ then } S_1 \texttt{ else } S_2\rangle} \tag{3}$$

For this exercise, you will prove the following representative inductive case:

$$\forall S \in \texttt{Stmt}.\forall E, E' \in \texttt{Var} \mapsto \mathbb{Z}.\langle E, \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2\rangle \Downarrow E' \iff \langle E, \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2\rangle \to^* \langle E', \texttt{skip}\rangle$$