Lecture 5: Data-Flow Analysis Examples

17-355/17-665/17-819: Program Analysis

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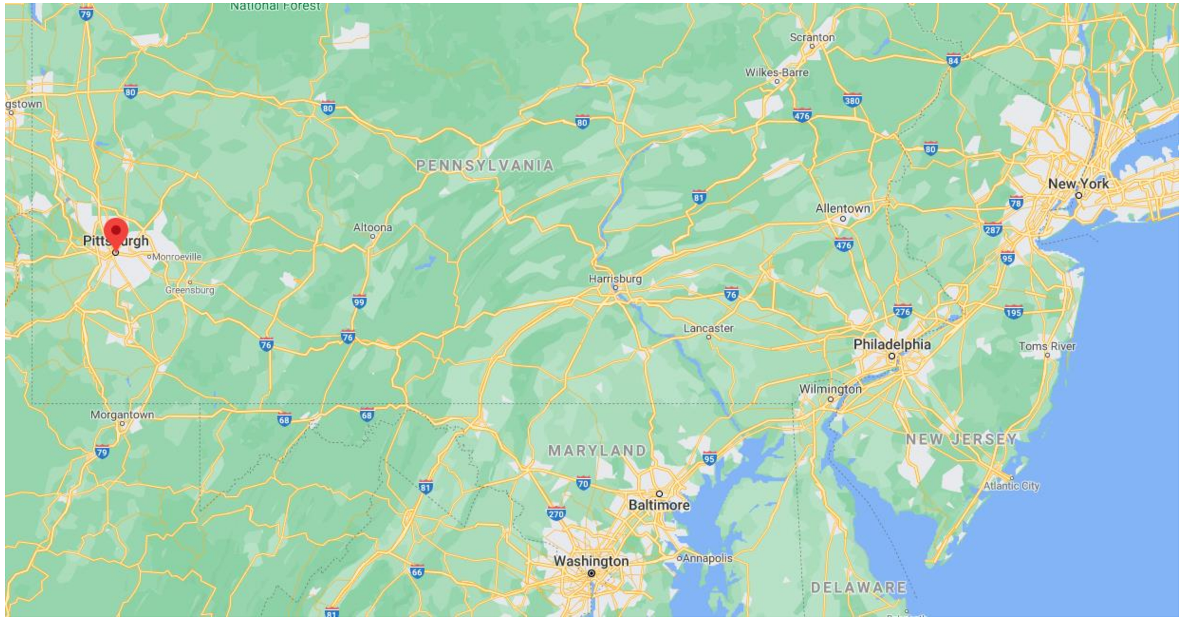
* Course materials developed with Jonathan Aldrich and Claire Le Goues

Trivia

"You are here" maps don't lie - Banach mapping theorem



What mathematical concept is common to both these facts?

Python:

```
exec(s:='print("exec(s:=%r)"%s)')
```

Review: Data-Flow Analysis

- a lattice $(L, \sqsubseteq)$
- an abstraction function α
- a flow function f
- initial dataflow analysis assumptions, σ_0

Review: Kildall's Algorithm

```
worklist = ∅
for  Node n in cfg
    input[n] = output[n] = ⊥
    add n to worklist
input[0] = initialDataflowInformation

while worklist is not empty
    take a Node n off the worklist
    output[n] = flow(n, input[n])
    for Node j in succs(n)
          newInput = input[j] ⊔ output[n]
          if newInput ≠ input[j]
                input[j] = newInput
                add j to worklist
```

Review: What order to process worklist nodes in?

- Random? Queue? Stack?
- Any order is valid (!!)
- Some orders are better in practice
  - Topological sorts are nice
  - Explore loops inside out
  - Reverse postorder!

Examples!! Classic Data-Flow Analyses

- Zero Analysis
- Integer Sign Analysis
- Constant Propagation
- Reaching Definitions
- Live Variables Analysis

- Available Expressions
- Very Busy Expressions
- …

Integer Sign Analysis

- Extension of Zero Analysis to track integers zero, less-than-zero, greater-than-zero, or ¯_(ツ)_/¯ (unknown).
- **Q**: Why do we care about sign?
- **Exercise 1**: What would the lattice for simple Sign Analysis look like?

Integer Sign Analysis

- Extension of simple Sign Analysis to track when x<0, x<=0, x=0, x>=0, x>0, x!=0, or unknown (¯_(ツ)_/¯).
- **Q**: Why do we care about all these cases?
- **Exercise 2**: How would the lattice for precise Sign Analysis look?

Constant Propagation

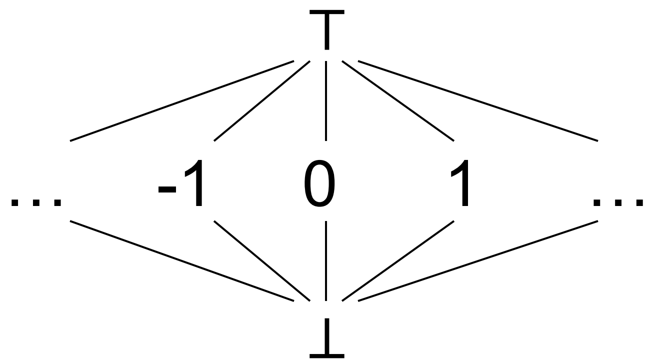
- Extension of Zero / Sign Analysis to track exact values of variables, if they are **constant** at a given program point (across all paths).
- E.g. x is 42 at line 10
- **Q**: Why is this useful?

Constant Propagation

$$\sigma \in Var \to L_{CP}$$

$$L_{CP} \text{ is } \mathbb{Z} \cup \{\top, \bot\}$$

$$\forall l \in L_{CP} : \bot \sqsubseteq l \wedge l \sqsubseteq \top$$



Constant Propagation

$$
\begin{array}{rcl}
\sigma & \in & \mathit{Var} \to L_{CP} \\
\sigma_1 \sqsubseteq_{lift} \sigma_2 & \mathit{iff} & \forall x \in \mathit{Var} : \sigma_1(x) \sqsubseteq \sigma_2(x) \\
\sigma_1 \sqcup_{lift} \sigma_2 & = & \{x \mapsto \sigma_1(x) \sqcup \sigma_2(x) \mid x \in \mathit{Var}\} \\
\top_{lift} & = & \{x \mapsto \top \mid x \in \mathit{Var}\} \\
\bot_{lift} & = & \{x \mapsto \bot \mid x \in \mathit{Var}\} \\
\alpha_{CP}(n) & = & n \\
\alpha_{lift}(E) & = & \{x \mapsto \alpha_{CP}(E(x)) \mid x \in \mathit{Var}\} \\
\sigma_0 & = & \top_{lift}
\end{array}
$$

Constant Propagation

$$f_{CP}[\![x := n]\!](\sigma) =$$

$$f_{CP}[\![x := y]\!](\sigma) =$$

$$f_{CP}[\![x := y \; op \; z]\!](\sigma) =$$

$$f_{CP}[\![\text{goto } n]\!](\sigma) =$$

$$f_{CP}[\![\text{if } x = 0 \text{ goto } n]\!]_T(\sigma) =$$

$$f_{CP}[\![\text{if } x = 0 \text{ goto } n]\!]_F(\sigma) =$$

$$f_{CP}[\![\text{if } x < 0 \text{ goto } n]\!](\sigma) =$$

Constant Propagation

$$f_{CP}[\![x := n]\!](\sigma) = \sigma[x \mapsto \alpha_{CP}(n)]$$

$$f_{CP}[\![x := y]\!](\sigma) = \sigma[x \mapsto \sigma(y)]$$

$$f_{CP}[\![x := y \; op \; z]\!](\sigma) = \sigma[x \mapsto \sigma(y) \; op_{lift} \; \sigma(z)]$$

where $n \; op_{lift} \; m = n \; op \; m$

and $n \; op_{lift} \; \bot = \bot$ (and symmetric)

and $n \; op_{lift} \; \top = \top$ (and symmetric)

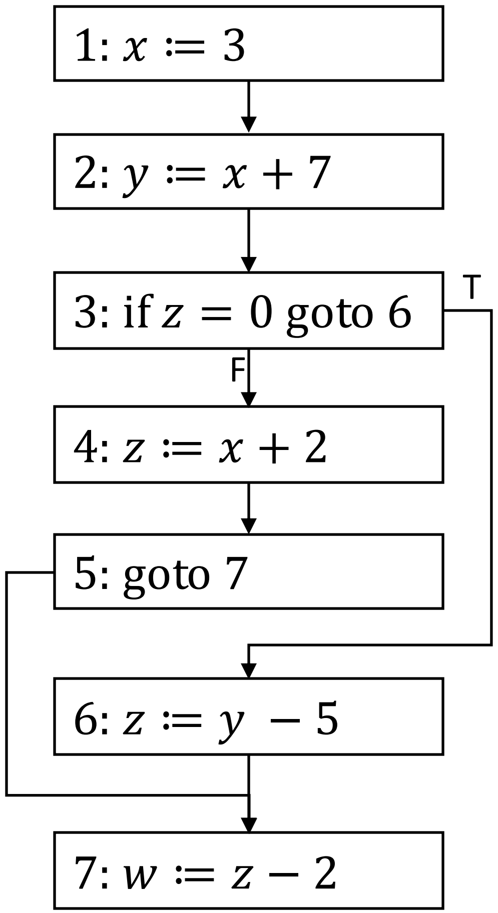
$$f_{CP}[\![\text{goto } n]\!](\sigma) = \sigma$$

$$f_{CP}[\![\text{if } x = 0 \text{ goto } n]\!]_T(\sigma) = \sigma[x \mapsto 0]$$

$$f_{CP}[\![\text{if } x = 0 \text{ goto } n]\!]_F(\sigma) = \sigma$$

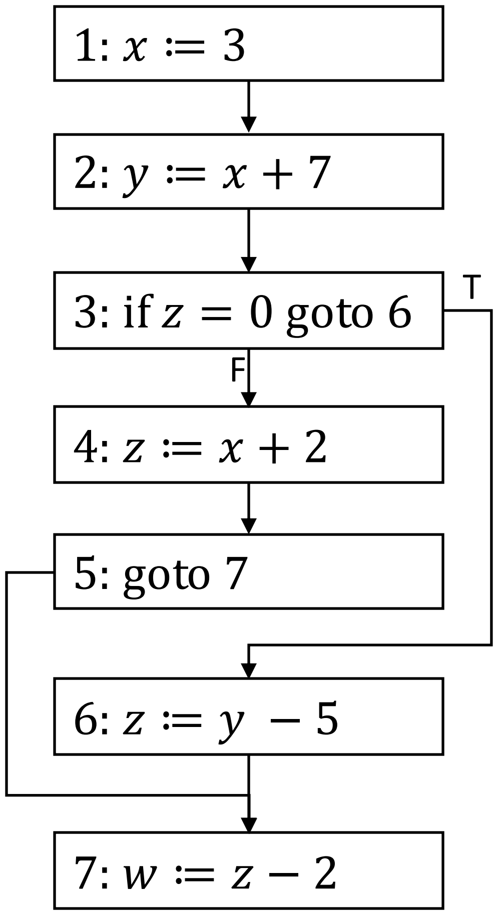
$$f_{CP}[\![\text{if } x < 0 \text{ goto } n]\!](\sigma) = \sigma$$

Constant Propagation



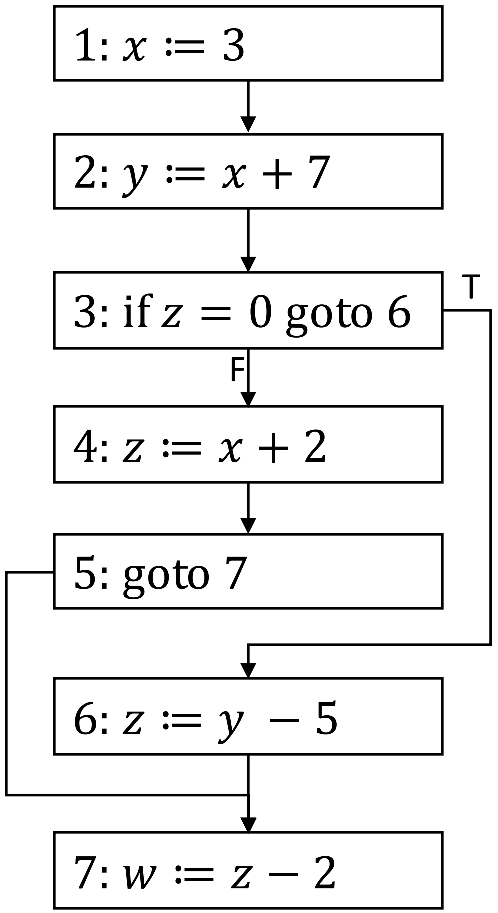
$$
\begin{array}{ll}
1: & x := 3 \\
2: & y := x + 7 \\
3: & \text{if } z = 0 \text{ goto } 6 \\
4: & z := x + 2 \\
5: & \text{goto } 7 \\
6: & z := y - 5 \\
7: & w := z - 2
\end{array}
$$

Constant Propagation



stmt	worklist	x	y	z	w

Constant Propagation



stmt	worklist	x	y	z	w
0	1,2,3,4,5,6,7	$\top$	$\top$	$\top$	$\top$
1	2,3,4,5,6,7	3	$\top$	$\top$	$\top$
2	3,4,5,6,7	3	10	$\top$	$\top$
3	4,5,6,7	3	10	$0_T, \top_F$	$\top$
4	5,6,7	3	10	5	$\top$
5	6,7	3	10	5	$\top$
6	7	3	10	5	$\top$
7	$\varnothing$	3	10	5	3

Reaching Definitions

- Where might a variable have last been defined?
  - Equivalent: what definitions of a variable *reach* this program point?
  - E.g. At line 7, the value of x was last obtained from assignments at lines 2 and 3.
- Lots of applications in compilers ("def-use chains")

- Let DEFS = set of all definitions
  - e.g. $\{x_1, x_2, y_3\}$

Reaching Definitions

$$
\begin{array}{rcl}
\sigma & \in & \mathcal{P}^{\textsf{DEFS}} \\
\sigma_1 \sqsubseteq \sigma_2 & \textit{iff} & \\
\sigma_1 \sqcup \sigma_2 & = & \\
\top & = & \\
\bot & = & \\
\sigma_0 & = &
\end{array}
$$

Reaching Definitions

$$
\begin{array}{rcl}
\sigma & \in & \mathcal{P}^{\text{DEFS}} \\
\sigma_1 \sqsubseteq \sigma_2 & \textit{iff} & \sigma_1 \subseteq \sigma_2 \\
\sigma_1 \sqcup \sigma_2 & = & \sigma_1 \cup \sigma_2 \\
\top & = & \text{DEFS} \\
\bot & = & \emptyset \\
\sigma_0 & = & \emptyset
\end{array}
$$

Reaching Definitions

$$f_{RD}[\![I]\!](\sigma) \quad = $$

Reaching Definitions

$$f_{RD}[\![I]\!](\sigma) = \sigma - KILL_{RD}[\![I]\!] \cup GEN_{RD}[\![I]\!]$$

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$$\text{KILL}_{RD}[\![n\text{: } x := ...]\!] =$$

$$\text{KILL}_{RD}[\![I]\!] =$$

$$\text{GEN}_{RD}[\![n\text{: } x := ...]\!] =$$

$$\text{GEN}_{RD}[\![I]\!] =$$

Reaching Definitions

$$f_{RD}[\![I]\!](\sigma) = \sigma - KILL_{RD}[\![I]\!] \cup GEN_{RD}[\![I]\!]$$

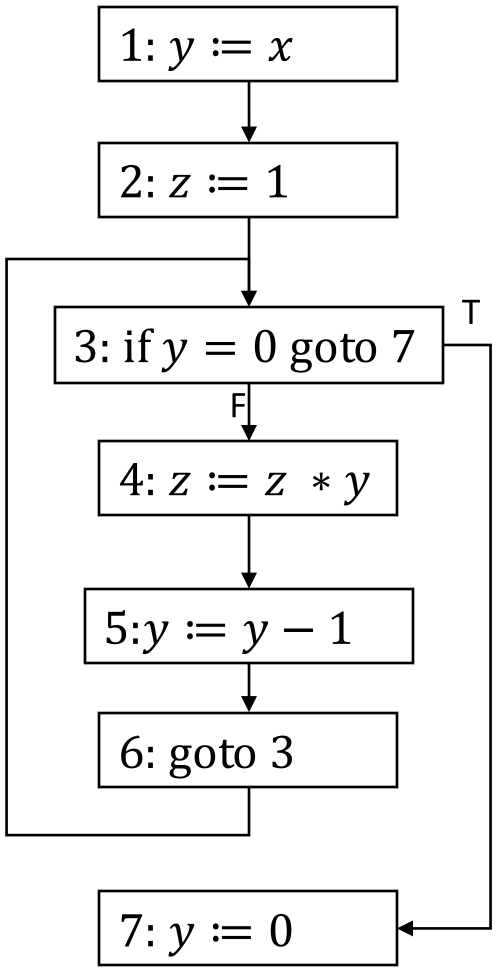
$$\text{KILL}_{RD}[\![n\text{: } x := ...]\!] = \{x_m \mid x_m \in \mathsf{DEFS}(x)\}$$

$$\text{KILL}_{RD}[\![I]\!] = \varnothing \quad \text{if } I \text{ is not an assignment}$$

$$\text{GEN}_{RD}[\![n\text{: } x := ...]\!] = \{x_n\}$$

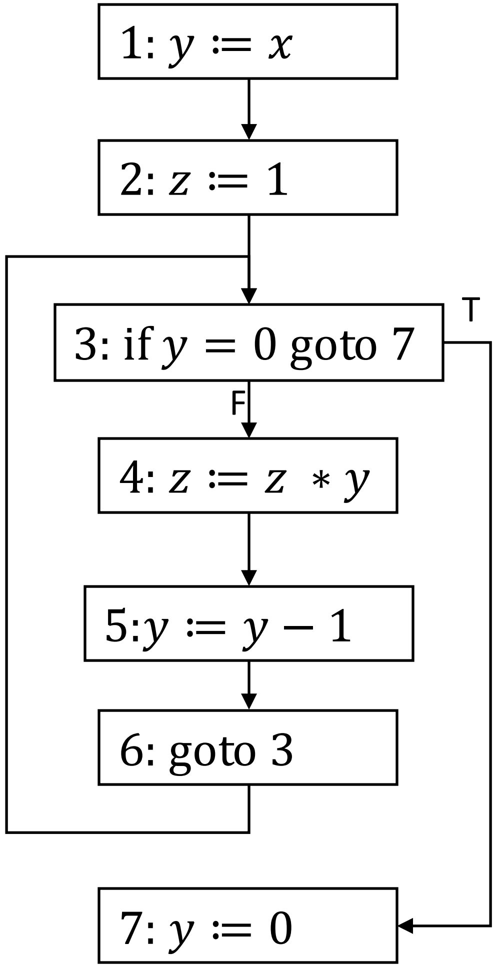
$$\text{GEN}_{RD}[\![I]\!] = \varnothing \quad \text{if } I \text{ is not an assignment}$$

Reaching Definitions



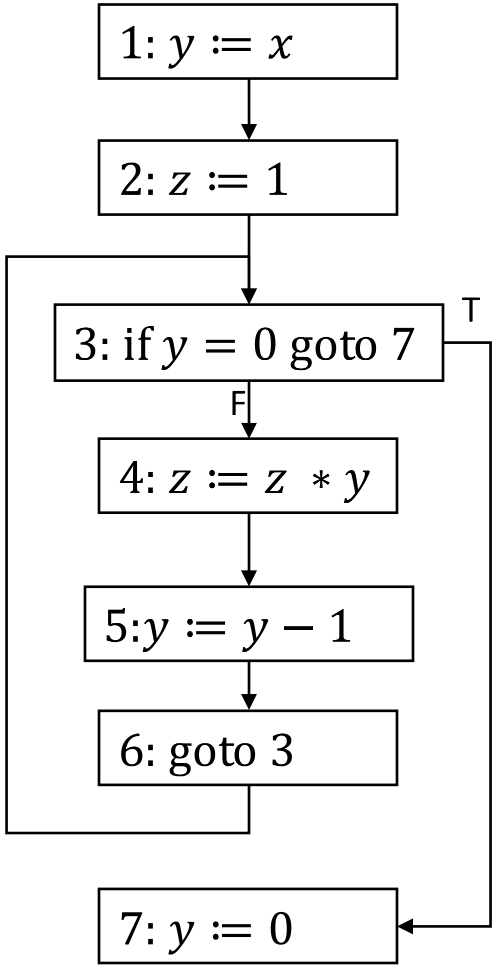
$$
\begin{aligned}
1 &: \quad y := x \\
2 &: \quad z := 1 \\
3 &: \quad \text{if } y = 0 \text{ goto } 7 \\
4 &: \quad z := z * y \\
5 &: \quad y := y - 1 \\
6 &: \quad \text{goto } 3 \\
7 &: \quad y := 0
\end{aligned}
$$

Reaching Definitions



stmt	worklist	defs

Reaching Definitions



stmt	worklist	defs
0	1,2,3,4,5,6,7	$\varnothing$
1	2,3,4,5,6,7	$\{y_1\}$
2	3,4,5,6,7	$\{y_1, z_1\}$
3	4,5,6,7	$\{y_1, z_1\}$
4	5,6,7	$\{y_1, z_4\}$
5	6,7	$\{y_5, z_4\}$
6	**3**,7	$\{y_5, z_4\}$
3	**4**,7	$\{y_1, y_5, z_1, z_4\}$
4	**5**,7	$\{y_1, y_5, z_4\}$
5	7	$\{y_5, z_4\}$
7	$\varnothing$	$\{y_7, z_1, z_4\}$

Live Variables

- Which variables will be used in the future (are “live”)?
- E.g. x is live at line 7 because it’s current value will be used at line 10.

- Another set-based analysis (like *reaching definitions*).
- Data-flow values propagate *backwards* !!!

Live Variables

$$\sigma \in \mathcal{P}^{\mathsf{Var}}$$

$$\sigma_1 \sqsubseteq \sigma_2 \quad \textit{iff}$$

$$\sigma_1 \sqcup \sigma_2 =$$

$$\top =$$

$$\bot =$$

Live Variables

$$
\begin{array}{rcl}
\sigma & \in & \mathcal{P}^{\mathsf{Var}} \\
\sigma_1 \sqsubseteq \sigma_2 & \textit{iff} & \sigma_1 \subseteq \sigma_2 \\
\sigma_1 \sqcup \sigma_2 & = & \sigma_1 \cup \sigma_2 \\
\top & = & \mathsf{Var} \\
\bot & = & \emptyset
\end{array}
$$

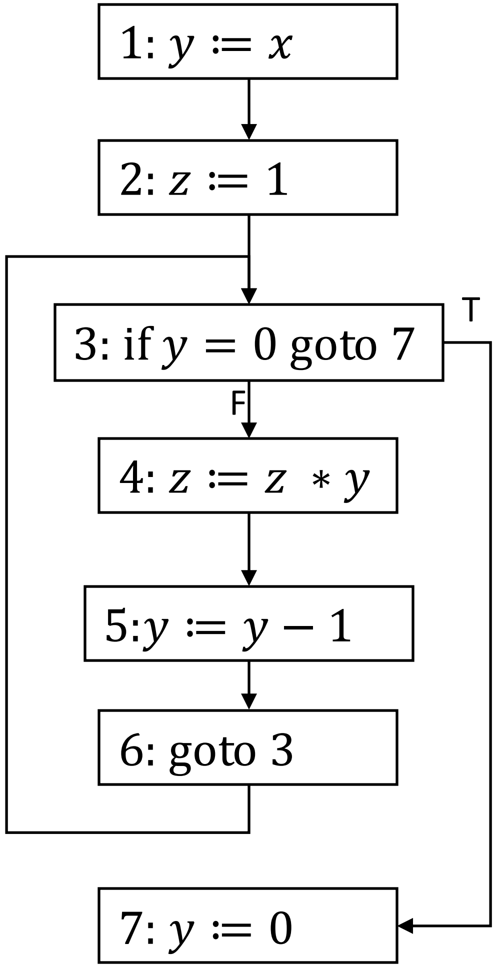
Live Variables

Flow functions map backward! (out --> in)

$$\text{KILL}_{LV}[\![I]\!] \quad = \{x \mid I \text{ defines } x\}$$

$$\text{GEN}_{LV}[\![I]\!] \quad = \{x \mid I \text{ uses } x\}$$

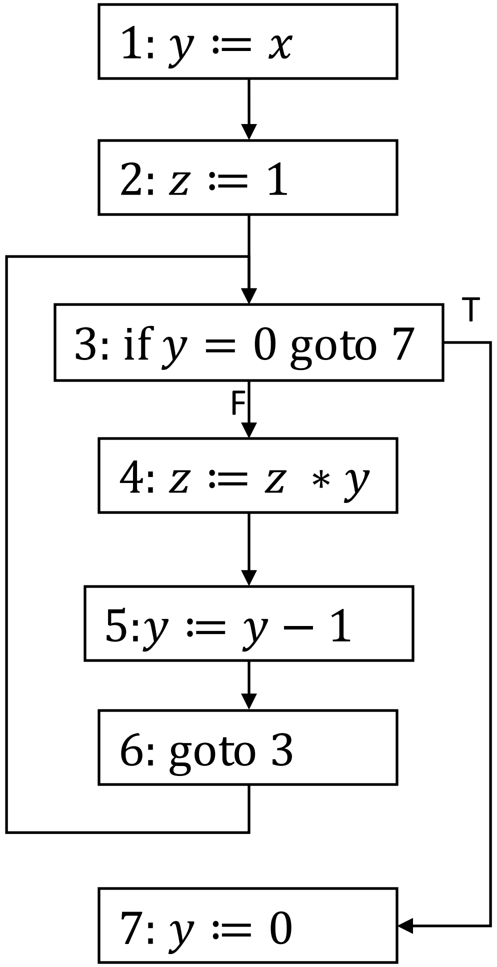
Live Variables



Assume result is in z

$$
\begin{array}{ll}
1: & y := x \\
2: & z := 1 \\
3: & \text{if } y = 0 \text{ goto } 7 \\
4: & z := z * y \\
5: & y := y - 1 \\
6: & \text{goto } 3 \\
7: & y := 0
\end{array}
$$

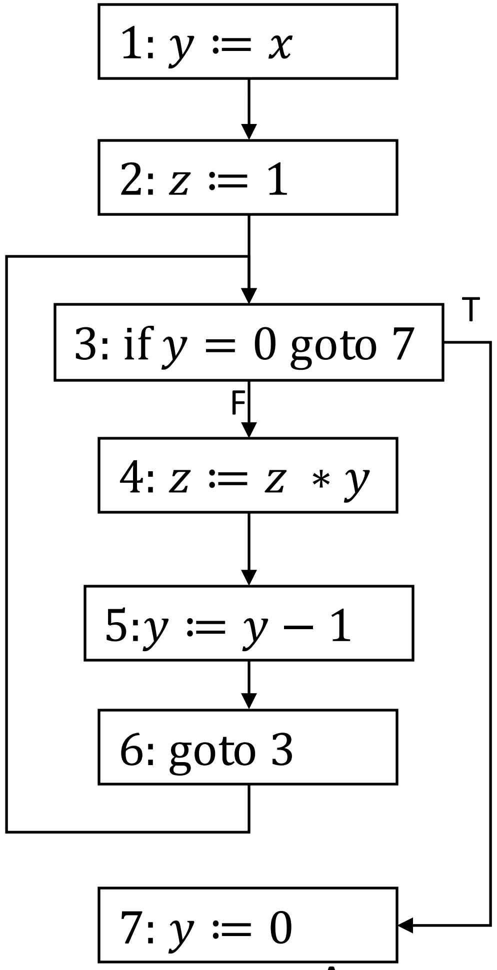
Live Variables



Assume result is in z

stmt	worklist	live

Live Variables



Assume result is in z

stmt	worklist	live
end	7,3,6,5,4,2,1	$\{z\}$
7	3,6,5,4,2,1	$\{z\}$
3	6,5,4,2,1	$\{z, y\}$
6	5,4,2,1	$\{z, y\}$
5	4,2,1	$\{z, y\}$
4	**3**,2,1	$\{z, y\}$
3	2,1	$\{z, y\}$
2	1	$\{y\}$
1	$\emptyset$	$\{x\}$