Dataflow Analysis Complexity and Correctness

The termination of the worklist algorithm for dataflow analysis relies on two conditions: the dataflow lattice having finite height and the flow functions being monotonic. A flow function f is monotonic iff $\sigma_1 \sqsubseteq \sigma_2$ implies $f(\sigma_1) \sqsubseteq f(\sigma_2)$ for all σ_1, σ_2.

The correctness of a dataflow analysis depends on the local soundness of its flow functions. For every program configuration c_i in the trace of a program P, a flow function f is locally sound iff $(P \vdash c_i \rightsquigarrow c_{i+1}$ and $\alpha(c_i) \sqsubseteq \sigma_{n_i}$ and $f[\![P[n_i]]\!](\sigma_{n_i}) = \sigma_{n_{i+1}}) \Rightarrow \alpha(c_{i+1}) \sqsubseteq \sigma_{n_{i+1}}$.

Exercises

These exercises prove properties of parity analysis. Assume the following:

- A lattice $(L, \sqsubseteq)$ where $L = \{\top, O, V, \bot\}$ and $\bot \sqsubseteq \{O, V\} \sqsubseteq \top$, $O \sqcup V = \top$
- An abstraction function $\alpha : \mathbb{Z} \mapsto L$, defined as follows:

$$\alpha(n) = \begin{cases} V \text{ when } n \text{ is an even integer } (n \in \{2k : k \in \mathbb{Z}\}) \\ O \text{ when } n \text{ is an odd integer } (n \in \{2k+1 : k \in \mathbb{Z}\}) \end{cases}$$

- a flow function f_P
- initial dataflow analysis assumptions σ_0, in this case σ_0 maps all variables' initial states to $\top$.

1. Disprove the local soundness of the incorrect flow function $f_P[\![a := b]\!](\sigma) = \sigma[a \mapsto O]$

For the next questions, use the following (correct) flow function for parity analysis:

$$f_P[\![a := b * c]\!](\sigma) = \begin{cases} \sigma[a \mapsto \bot] & \text{if } \sigma(b) = \bot \vee \sigma(c) = \bot \\ \sigma[a \mapsto O] & \text{if } \sigma(b) = O \wedge \sigma(c) = O \\ \sigma[a \mapsto V] & \text{if } (\sigma(b) = V \wedge \sigma(c) \neq \bot) \vee (\sigma(b) \neq \bot \wedge \sigma(c) = V) \\ \sigma[a \mapsto \top] & \text{if } (\sigma(b) = \top \wedge \sigma(c) \notin \{V, \bot\}) \vee (\sigma(b) \notin \{V, \bot\} \wedge \sigma(c) = \top) \end{cases}$$

2. Prove the monotonicity of $f_P[\![a := b*c]\!](\sigma)$ for the case $(\sigma_1(b) = V \wedge \sigma_1(c) \neq \bot) \vee (\sigma_1(b) \neq \bot \wedge \sigma_1(c) = V)$

3. Prove the local soundness of $f_P[\![a := b * c]\!](\sigma)$