Lecture 25: Review of Program Analysis

17-355/17-665/17-819: Program Analysis

Rohan Padhye

April 28, 2022

* Course materials developed with Jonathan Aldrich and Claire Le Goues

What is this course about?

- Program analysis is the systematic examination of a program to determine its properties.
- From 30,000 feet, this requires:
  - Precise program representations
  - Tractable, systematic ways to reason over those representations.
- ~~We will learn~~: What we learned:
  - How to unambiguously define the meaning of a program, and a programming language.
  - How to prove theorems about the behavior of particular programs.
  - How to use, build, and extend tools that do the above, automatically.

What is this course about?

- Program analysis is *the systematic examination of a program to determine its properties*.
- Principal techniques:
  - **Dynamic:**
    - **Testing:** Direct execution of code on test data in a controlled environment.
    - **Analysis:** Tools extracting data from test runs.
  - **Static:**
    - **Inspection:** Human evaluation of code, design documents (specs and models), modifications.
    - **Analysis:** Tools reasoning about the program without executing it.
  - ...and their combination.

The Bad News: Rice's Theorem

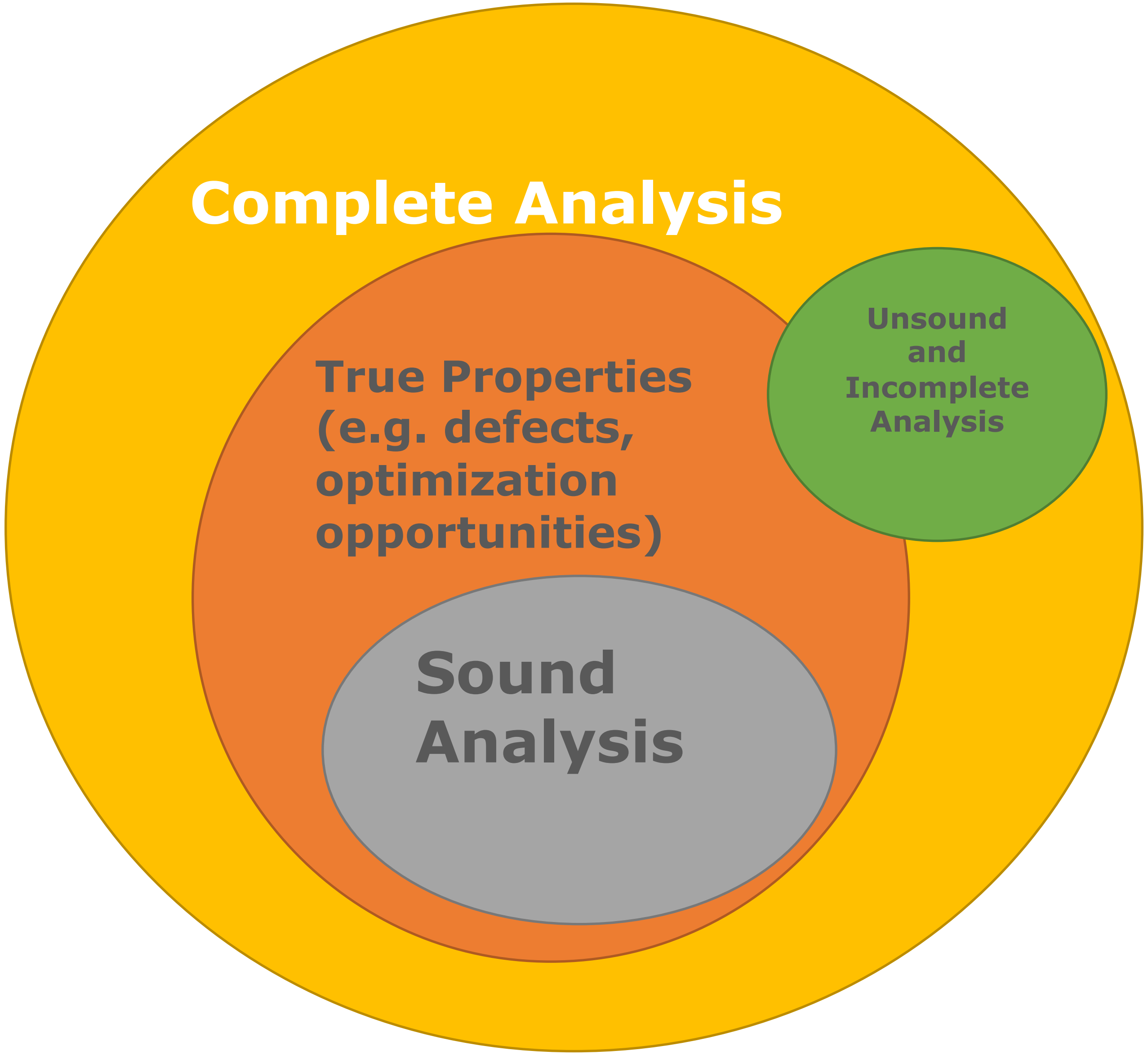
"Any nontrivial property about the language recognized by a Turing machine is undecidable.“

Henry Gordon Rice, 1953

Soundness and Completeness

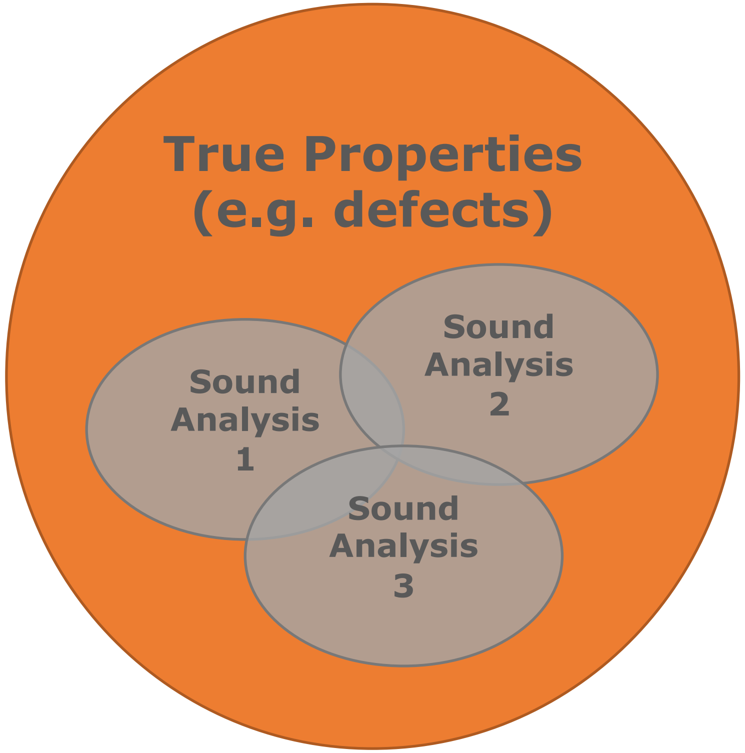
- An analysis is "sound" if every claim it makes is true
- An analysis is "complete" if it makes every true claim

- Soundness/Completeness correspond to under/over-approximation depending on context.
  - E.g. compilers and verification tools treat "soundness" as over-approximation since they make claims over all possible inputs
  - E.g. code quality tools often treat "sound" analyses as under-approximation because they make claims about existence of bugs


Complete Analysis
True Properties (e.g. defects, optimization opportunities)
Unsound and Incomplete Analysis
Sound Analysis

Soundness and Completeness Tradeoffs

- Sound + Complete is impossible in general (Rice's theorem)
- Most practical tools attempt to be either sound or complete for some specific application, using approximation
- Multiple classes of sound/complete techniques may exist, with trade-offs for accuracy and performance.
- Program analysis is a rich field because of the constant and never-ending battle to balance these trade-offs with ever-increasing software complexity



Fundamental concepts

- Abstraction
  - Elide details of a specific implementation.
  - Capture semantically relevant details; ignore the rest.
- The importance of semantics.
  - We prove things about analyses with respect to the semantics of the underlying language.
- Program proofs as inductive invariants.
- Implementation
  - You do not understand analysis until you have written several.

What you were supposed to get

- Beautiful and elegant theory
  - Mostly discrete mathematics, symbolic reasoning, inductive proofs
  - This is traditionally a "white-board" course [using slides while we're on Zoom]
- Build awesome tools
  - Engineering of program analyses, compilers, and bug finding tools make great use of many fundamental ideas from computer science and software engineering
- New way to think about programs
  - Representations, control/data-flow, input state space
- Appreciate the limits and achievements in the space
  - What tools are *impossible* to build?
  - What tools are *impressive* that they exist at all?
  - When is it appropriate to use a particular analysis tool versus another?
  - How to interpret the results of a program analysis tool?

The WHILE language – Example program

```
y := x;
z := 1;
if y > 0 then
  while y > 1 do
    z := z * y;
    y := y – 1
else
  skip
```

- Sample program computes z = *x!* using *y* as a temp variable.
- WHILE uses assignment statements, if-then-else, while loops.
- All vars are integers.
- Expressions only arithmetic (for vars) or relational (for conditions).
- No I/O statements. Inputs and outputs are implicit.
  - Later on, we may use extensions with explicit `read x` and `print x`.

WHILE abstract syntax

Concrete syntax is similar, but adds things like (parentheses) for disambiguation during parsing

- Categories:
  - $S \in$ **Stmt** statements
  - $a \in$ **Aexp** arithmetic expressions
  - $x, y \in$ **Var** variables
  - $n \in$ **Num** number literals
  - $P \in$ **BExp** boolean predicates
  - $l \in$ **labels** statement addresses (line numbers)
- Syntax:
  - $S ::= x := a \mid \texttt{skip} \mid S_1 ; S_2$
    $\mid \texttt{if}\ P\ \texttt{then}\ S_1\ \texttt{else}\ S_2 \mid \texttt{while}\ P\ \texttt{do}\ S$
  - $a ::= x \mid n \mid a_1\ op_a\ a_2$
  - $op_a ::= + \mid - \mid * \mid / \mid \ldots$
  - $P ::= \texttt{true} \mid \texttt{false} \mid \texttt{not}\ P \mid P_1\ op_b\ P_2 \mid a1\ op_r\ a2$
  - $op_b ::= \texttt{and} \mid \texttt{or} \mid \ldots$
  - $op_r ::= < \mid \leq \mid = \mid > \mid \geq \mid \ldots$

Our first static analysis: AST walking

- One way to find “bugs” is to walk the AST, looking for particular patterns.
  - Traverse the AST, look for nodes of a particular type
  - Check the neighborhood of the node for the pattern in question.
  - Basically, a glorified “grep” that knows about the syntax but not semantics of a language.

CodeQL

- A language for querying code. Developed by GitHub.
- Supports many common languages.
- Library of common programming patterns and optimizations.

CodeQL queries 1.23

Dashboard / Java queries

Inefficient empty string test

Created by Documentation team, last modified on Mar 28, 2019

Name: Inefficient empty string test

Description: Checking a string for equality with an empty string is inefficient.

ID: java/inefficient-empty-string-test

Kind: problem

Severity: recommendation

Precision: high

Query: InefficientEmptyStringTest.ql › Expand source

When checking whether a string `s` is empty, perhaps the most obvious solution is to write something like `s.equals("")` (or `"".equals(s)`). However, this actually carries a fairly significant overhead, because `String.equals` performs a number of type tests and conversions before starting to compare the content of the strings.

Recommendation

The preferred way of checking whether a string `s` is empty is to check if its length is equal to zero. Thus, the condition is `s.length() == 0`. The `length` method is implemented as a simple field access, and so should be noticeably faster than calling `equals`.

Note that in Java 6 and later, the `String` class has an `isEmpty` method that checks whether a string is empty. If the codebase does not need to support Java 5, it may be better to use that method instead.

https://help.semmle.com/wiki/display/JAVA/Inefficient+empty+string+test

Operational Semantics of WHILE

- The meaning of WHILE expressions depend on the values of variables
  - What does x+5 mean? It depends on x.
  - If $x = 8$ at some point, we expect x+5 to mean 13
- The value of integer variables at a given moment is abstracted as a function:

$$E : Var \rightarrow Z$$

- We will augment our notation of big-step evaluation to include state:

$$\langle E, a \rangle \Downarrow n$$

- So, if $\{x \mapsto 8\} \in E$, then $\langle E, x + 5 \rangle \Downarrow 13$

Big-Step Semantics for WHILE expressions

$$\frac{}{\langle E, n\rangle \Downarrow n}\ \textit{big-int} \qquad \frac{}{\langle E, x\rangle \Downarrow E(x)}\ \textit{big-var}$$

$$\frac{\langle E, a_1\rangle \Downarrow n_1 \quad \langle E, a_2\rangle \Downarrow n_2}{\langle E, a_1 + a_2\rangle \Downarrow n_1 + n_2}\ \textit{big-add}$$

- Similarly for other arithmetic and boolean expressions

Big-Step Semantics for WHILE statements

$$\frac{\langle E, b\rangle \Downarrow \texttt{false}}{\langle E, \texttt{while}\ b\ \texttt{do}\ S\rangle \Downarrow E}\ \textit{big-whilefalse}$$

$$\frac{\langle E, b\rangle \Downarrow \texttt{true} \qquad \langle E, S; \texttt{while}\ b\ \texttt{do}\ S\rangle \Downarrow E'}{\langle E, \texttt{while}\ b\ \texttt{then}\ S\rangle \Downarrow E'}\ \textit{big-whiletrue}$$

Small-Step Semantics for WHILE statements

$$\frac{\langle E, S_1\rangle \rightarrow \langle E', S_1'\rangle}{\langle E, S_1; S_2\rangle \rightarrow \langle E', S_1'; S_2\rangle}\ \textit{small-seq-congruence}$$

$$\frac{}{\langle E, \texttt{skip}; S_2\rangle \rightarrow \langle E, S_2\rangle}\ \textit{small-seq}$$

Small-Step Semantics for WHILE statements

$$\frac{\langle E, b\rangle \rightarrow_b b'}{\langle E, \texttt{if } b \texttt{ then } S_1 \texttt{ else } S_2\rangle \rightarrow \langle E, \texttt{if } b' \texttt{ then } S_1 \texttt{ else } S_2\rangle}\ \textit{small-if-congruence}$$

$$\frac{}{\langle E, \texttt{if true then } S_1 \texttt{ else } S_2\rangle \rightarrow \langle E, S_1\rangle}\ \textit{small-iftrue}$$

Proofs by Structural Induction

Example. Let $L(a)$ be the number of literals and variable occurrences in some expression a and $O(a)$ be the number of operators in a. Prove by induction on the structure of a that $\forall a \in \texttt{Aexp} \, . \, L(a) = O(a) + 1$:

Base cases:

- Case $a = n$. $L(a) = 1$ and $O(a) = 0$
- Case $a = x$. $L(a) = 1$ and $O(a) = 0$

Inductive case 1: Case $a = a_1 + a_2$

- By definition, $L(a) = L(a_1) + L(a_2)$ and $O(a) = O(a_1) + O(a_2) + 1$.
- By the induction hypothesis, $L(a_1) = O(a_1) + 1$ and $L(a_2) = O(a_2) + 1$.
- Thus, $L(a) = O(a_1) + O(a_2) + 2 = O(a) + 1$.

The other arithmetic operators follow the same logic.

Proofs by Structural Induction

- Prove that WHILE is *deterministic*. That is, if the program terminates, it evaluates to a unique value.

$$
\begin{array}{lll}
\forall a \in \texttt{Aexp} . & \forall E . \forall n, n' \in \mathbb{N} . & \langle E, a \rangle \Downarrow n \wedge \langle E, a \rangle \Downarrow n' \Rightarrow n = n' \\
\forall P \in \texttt{Bexp} . & \forall E . \forall b, b' \in \mathcal{B} . & \langle E, P \rangle \Downarrow b \wedge \langle E, P \rangle \Downarrow b' \Rightarrow b = b' \\
\forall S . & \forall E, E', E'' . & \langle E, S \rangle \Downarrow E' \wedge \langle E, S \rangle \Downarrow E'' \Rightarrow E' = E''
\end{array}
$$

Rule for while is recursive; doesn't depend only on subexpressions

- Can prove for expressions via induction over syntax, but not for statements.
- But there's still a way.

To prove: $\forall S \,.\quad \forall E, E', E''\,.\quad \langle E, S\rangle \Downarrow E' \wedge \langle E, S\rangle \Downarrow E'' \Rightarrow E' = E''$

Structural Induction over Derivations

Base case: the one rule with no premises, `skip`:

let $D :: \langle E, S\rangle \Downarrow E'$, and let $D' :: \langle E, S\rangle \Downarrow E''$

$$D ::= \overline{\langle E, \mathtt{skip}\rangle \Downarrow E}$$

By inversion, the last rule used in D' (which, again, produced E'') must also have been the rule for `skip`. By the structure of the `skip` rule, we know $E'' = E$.

Inductive cases: We need to show that the property holds when the last rule used in D was each of the possible non-skip WHILE commands. I will show you one representative case; the rest are left as an exercise. If the last rule used was the `while-true` statement:

$$D ::= \frac{D_1 :: \langle E, b\rangle \Downarrow \mathtt{true} \quad D_2 :: \langle E, S\rangle \Downarrow E_1 \quad D_3 :: \langle E_1, \mathtt{while}\ b\ \mathtt{do}\ S\rangle \Downarrow E'}{\langle E, \mathtt{while}\ b\ \mathtt{do}\ S\rangle \Downarrow E'}$$

Pick arbitrary E'' such that $D' :: \langle E, \mathtt{while}\ b\ \mathtt{do}\ S\rangle \Downarrow E''$

By inversion, D' must use either the `while-true` or the `while-false` rule. However, having proved that boolean expressions are deterministic (via induction on syntax), and given that D contains the judgment $\langle E, b\rangle \Downarrow \mathtt{true}$, we know that D' cannot be the `while-false` rule, as otherwise it would have to contain a contradicting judgment $\langle E, b\rangle \Downarrow \mathtt{false}$.

So, we know that D' is also using `while-true` rule. In its derivation, D' must also have subderivations $D'_2 :: \langle E, S\rangle \Downarrow E'_1$ and $D'_3 :: \langle E'_1, \mathtt{while}\ b\ \mathtt{do}\ S\rangle \Downarrow E''$. By the induction hypothesis on D_2 with D'_2, we know $E_1 = E'_1$. Using this result and the induction hypothesis on D_3 with D'_3, we have $E'' = E'$.

Data-Flow Analysis

Computes universal properties about program state at specific program points. (e.g. will *x* be zero at line 7?)

- About program state
  - About data store (e.g. variables, heap memory)
  - Not about control (e.g. termination, performance)
- At program points
  - Statically identifiable (e.g. line 7, or when foo() calls bar())
  - Not dynamically computed (E.g. when x is 12 or when foo() is invoked 12 times)
- Universal
  - Reasons about all possible executions (always/never/maybe)
  - Not about specific program paths (see: symbolic execution, testing)

WHILE3ADDR: An Intermediate Representation

- Simpler, more uniform than WHILE syntax
- Categories:
  - $I \in$ **Instruction** instructions
  - $x, y \in$ **Var** variables
  - $n \in$ **Num** number literals
- Syntax:
  - I ::= x := n | x := y | $x := y\ op\ z$
    | goto n / if x $\mathtt{op}_\mathtt{r}$ 0 goto n
  - op_a ::= + | - | * | / | …
  - op_r ::= < | ≤ | = | > | ≥ | ...
  - P ∈ **Num** → I

$$\frac{P(n) = x := m}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E[x \mapsto m], n+1 \rangle} \; \textit{step-const}$$

$$\frac{P[n] = x := y}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E[x \mapsto E(y)], n+1 \rangle} \; \textit{step-copy}$$

$$\frac{P(n) = x := y \; op \; z \quad E(y) \; \mathbf{op} \; E(z) = m}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E[x \mapsto m], n+1 \rangle} \; \textit{step-arith}$$

$$\frac{P(n) = \text{goto } m}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E, m \rangle} \; \textit{step-goto}$$

$$\frac{P(n) = \text{if } x \; op_r \; 0 \text{ goto } m \quad E(x) \; \mathbf{op_r} \; 0 = true}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E, m \rangle} \; \textit{step-iftrue}$$

$$\frac{P(n) = \text{if } x \; op_r \; 0 \text{ goto } m \quad E(x) \; \mathbf{op_r} \; 0 = false}{P \vdash \langle E, n \rangle \rightsquigarrow \langle E, n+1 \rangle} \; \textit{step-iffalse}$$

Classic Data-Flow Analyses

- Zero Analysis
- Integer Sign Analysis
- Constant Propagation
- Reaching Definitions
- Live Variables Analysis

- Available Expressions
- Very Busy Expressions
- …

Partial Order & Join on set L

$l_1 \sqsubseteq l_2 \quad : \quad l_1$ is at least as precise as l_2

reflexive: $\forall l : l \sqsubseteq l$
transitive: $\forall l_1, l_2, l_3 : l_1 \sqsubseteq l_2 \wedge l_2 \sqsubseteq l_3 \Rightarrow l_1 \sqsubseteq l_3$
anti-symmetric: $\forall l_1, l_2 : l_1 \sqsubseteq l_2 \wedge l_2 \sqsubseteq l_1 \Rightarrow l_1 = l_2$

$l_1 \sqcup l_2$: **join** or *least-upper-bound*… "most precise generalization"

L is a *join-semilattice* iff: $l_1 \sqcup l_2$ always exists and is unique $\forall l_1, l_2 \in L$

$\top$ ("top") is the maximal element

Fixed point of Flow Functions

$$(\sigma_0, \sigma_1, \sigma_2, \ldots, \sigma_n) \xrightarrow{f_Z} (\sigma'_0, \sigma'_1, \sigma'_2, \ldots, \sigma'_n)$$

$$\sigma'_0 = \sigma_0$$

$$\sigma'_1 = f_Z[\![x := 10]\!](\sigma_0)$$

$$\sigma'_2 = f_Z[\![y := 0]\!](\sigma_1)$$

$$\sigma'_3 = \sigma_2 \sqcup \sigma_7$$

$$\sigma'_4 = f_Z[\![\text{if } x = 10 \text{ goto } 7]\!]_F(\sigma_3)$$

$$\vdots$$

$$\sigma'_8 = f_Z[\![\text{if } x = 10 \text{ goto } 7]\!]_T(\sigma_3)$$

$$\sigma'_9 = f_Z[\![x := y]\!](\sigma_8)$$

Fixed point!

$$\boxed{(\sigma_0, \sigma_1, \sigma_2, \ldots, \sigma_n) = f_Z(\sigma_0, \sigma_1, \sigma_2, \ldots, \sigma_n)}$$

Correctness theorem:
If data-flow analysis is well designed*, then any fixed point of the analysis is sound.

Kildall's Algorithm

```
worklist = ∅
for  Node n in cfg
     input[n] = output[n] = ⊥
     add n to worklist
input[0] = initialDataflowInformation

while worklist is not empty
     take a Node n off the worklist
     output[n] = flow(n, input[n])
     for Node j in succs(n)
           newInput = input[j] ⊔ output[n]
           if newInput ≠ input[j]
                 input[j] = newInput
                 add j to worklist
```

Worklist Algorithm Terminates at Fixed Point

At the fixed point, we therefore have the following equations satisfied:

$$\begin{array}{rcl} \sigma_0 & \sqsubseteq & \sigma_1 \\ \forall i \in P : \left(\bigsqcup_{j \in \mathtt{preds}(i)} f[\![P[j]]\!](\sigma_j) \right) & \sqsubseteq & \sigma_i \end{array}$$

The worklist algorithm shown above computes a fixed point when it terminates. We can prove this by showing that the following loop invariant is maintained:

$$\forall i \,.\, (\exists j \in preds(i) \textit{ such that } f[\![P[j]]\!](\sigma_j) \not\sqsubseteq \sigma_i) \Rightarrow i \in \mathtt{worklist}$$

Program Traces and DataFlow Soundness

A trace T of a program P is a potentially infinite sequence $\{c_0, c_1, ...\}$ of program configurations, where $c_0 = E_0, 1$ is called the initial configuration, and for every $i \geqslant 0$ we have $P \vdash c_i \rightsquigarrow c_{i+1}$

The result $\langle \sigma_n \mid n \in P \rangle$ of a program analysis running or program P is sound iff, for all traces T of P, for all i such that $0 \leqslant i < length(T)$, $\alpha(c_i) \sqsubseteq \sigma_{n_i}$

Fixed Point Theorem

Theorem 2 (A fixed point of a locally sound analysis is globally sound)**.** *If a dataflow analysis's flow function f is monotonic and locally sound, and for all traces T we have $\alpha(c_0) \sqsubseteq \sigma_0$ where σ_0 is the initial analysis information, then any fixed point $\{\sigma_n \mid n \in P\}$ of the analysis is sound.*

Proof. To show that the analysis is sound, we must prove that for all program traces, every program configuration in that trace is correctly approximated by the analysis results. We consider an arbitrary program trace T and do the proof by induction on the program configurations $\{c_i\}$ in the trace.

Least Fixed Point (LFP)

The least fixed point solution of a composite flow function $\mathcal{F}$ is the fixed-point result Σ^* such that $\mathcal{F}(\Sigma^*) = \Sigma^*$ and $\forall \Sigma : (\mathcal{F}(\Sigma) = \Sigma) \Rightarrow (\Sigma^* \sqsubseteq \Sigma)$.

Merge Over Paths (MOP)

We first enumerate all paths π of the form $\pi = n_1, n_2, \ldots$ in the control-flow graph, where n_i are the instructions (nodes) in the path. For each such path π, we successively apply flow functions to form the sequence of tuples $\Pi = \langle \sigma_1, n_1 \rangle, \langle \sigma_2, n_2 \rangle, \ldots$ such that $\sigma_{\Pi_j} = f[\![P[n_{\Pi_j}]]\!](\sigma_{\Pi_{j-1}})$, where Π_j is the j-th tuple in the sequence and σ_0 is the initial data flow information. We then join over all σ values computed for an instruction i to get the MOP:

$$\mathrm{MOP}(i) = \bigsqcup \{\sigma \mid \langle \sigma, i \rangle \in \text{Some } \Pi \text{ for } P\}$$

The MOP solution is the most precise result if we consider all possible program paths through the CFG, even though it may be less precise than the optimal solution due to the consideration of infeasible paths. The MOP is computable when flow functions are *distributive* over join.

Distributivity A function f is *distributive* iff $f(\sigma_1) \sqcup f(\sigma_2) = f(\sigma_1 \sqcup \sigma_2)$

Reaching Definitions

$$f_{RD}[\![I]\!](\sigma) = \sigma - KILL_{RD}[\![I]\!] \cup GEN_{RD}[\![I]\!]$$

$$\text{KILL}_{RD}[\![n\colon\; x := \ldots]\!] = \{x_m \mid x_m \in \textsf{DEFS}(x)\}$$

$$\text{KILL}_{RD}[\![I]\!] = \varnothing \qquad \text{if } I \text{ is not an assignment}$$

$$\text{GEN}_{RD}[\![n\colon\; x := \ldots]\!] = \{x_n\}$$

$$\text{GEN}_{RD}[\![I]\!] = \varnothing \qquad \text{if } I \text{ is not an assignment}$$

Live Variables

Flow functions map backward! (out --> in)

$$
\begin{aligned}
\text{KILL}_{LV}[\![I]\!] &= \{x \mid I \text{ defines } x\} \\
\text{GEN}_{LV}[\![I]\!] &= \{x \mid I \text{ uses } x\}
\end{aligned}
$$

Constant Propagation

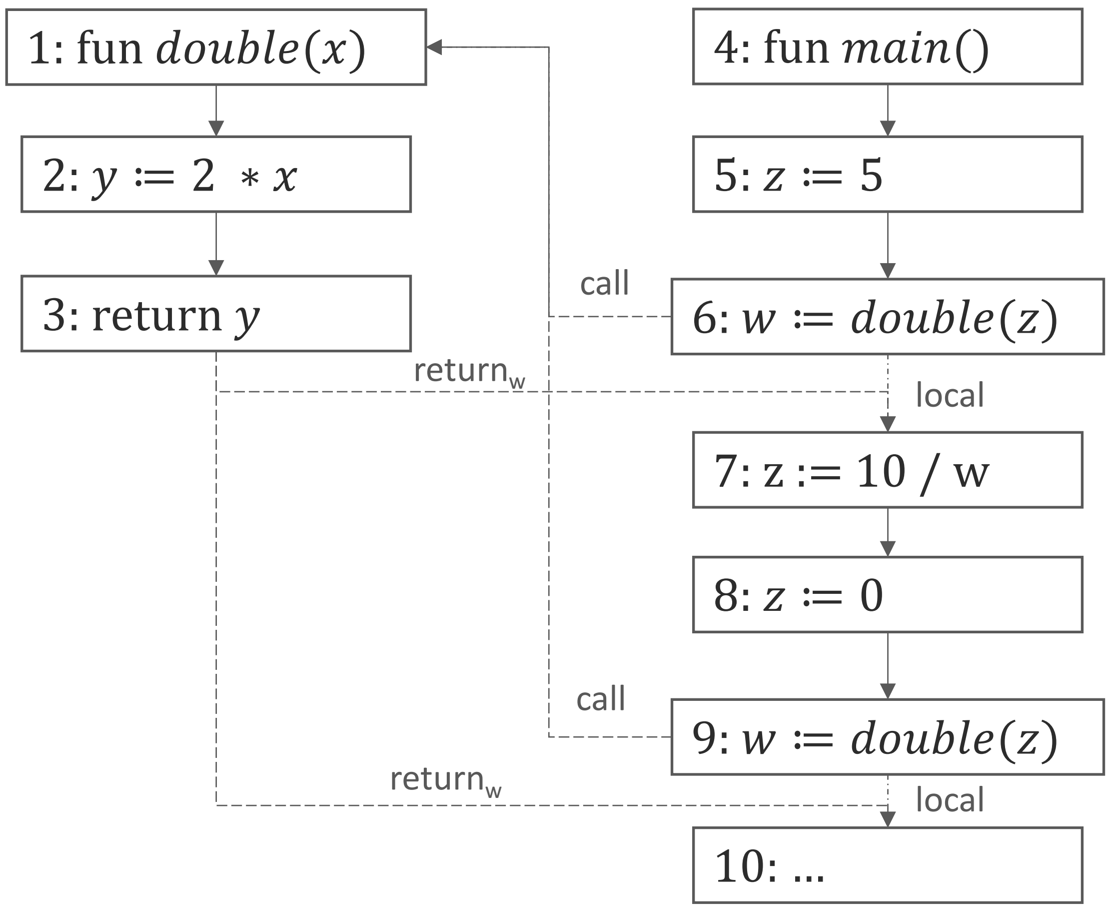
$$
\begin{array}{rcl}
\sigma & \in & Var \rightarrow L_{CP} \\
\sigma_1 \sqsubseteq_{lift} \sigma_2 & \textit{iff} & \forall x \in Var : \sigma_1(x) \sqsubseteq \sigma_2(x) \\
\sigma_1 \sqcup_{lift} \sigma_2 & = & \{x \mapsto \sigma_1(x) \sqcup \sigma_2(x) \mid x \in Var\} \\
\top_{lift} & = & \{x \mapsto \top \mid x \in Var\} \\
\bot_{lift} & = & \{x \mapsto \bot \mid x \in Var\} \\
\alpha_{CP}(n) & = & n \\
\alpha_{lift}(E) & = & \{x \mapsto \alpha_{CP}(E(x)) \mid x \in Var\} \\
\sigma_0 & = & \top_{lift}
\end{array}
$$

Extend WHILE3ADDR with functions

$$
\begin{array}{lcl}
F & ::= & \text{fun } f(x) \ \{ \ \overline{n : I} \ \} \\
I & ::= & \ldots \ | \ \text{return } x \ | \ y := f(x)
\end{array}
$$

$$
\begin{array}{ll}
1: & \text{fun } \mathit{double}(x) : \mathit{int} \\
2: & \quad y := 2 * x \\
3: & \quad \text{return } y \\
4: & \text{fun } \mathit{main}() : \mathit{void} \\
5: & \quad z := 0 \\
6: & \quad w := \mathit{double}(z)
\end{array}
$$

Interprocedural CFG



$$
\begin{array}{ll}
1: & \text{fun } double(x) : int \\
2: & \quad y := 2 * x \\
3: & \quad \text{return } y \\
4: & \text{fun } main() \\
5: & \quad z := 5 \\
6: & \quad w := double(z) \\
7: & \quad z := 10/w \\
8: & \quad z := 0 \\
9: & \quad w := double(z)
\end{array}
$$

$f_Z[\![x := g(y)]\!]_{local}(\sigma) = \sigma \setminus (\{x\} \cup Globals)$

$f_Z[\![x := g(y)]\!]_{call}(\sigma) = \{v \mapsto \sigma(v) \mid v \in Globals\} \cup \{formal(g) \mapsto \sigma(y)\}$

$f_Z[\![\text{return } y]\!]_{return_x}(\sigma) = \{v \mapsto \sigma(v) \mid v \in Globals\} \cup \{x \mapsto \sigma(y)\}$

Context-Sensitive Analysis Example

```
fun double(x) : int
    y := 2 * x
    return y

fun main()
    z := 5
    w := double(z)
    z := 10/w
    z := 0
    w := double(z)
```

Context	σ_{in}	σ_{out}
<main, T>	T	{w->Z, Z->Z}
<double, N>	{x->N}	{x->N, y->N}
<double, Z>	{x->Z}	{x->Z, y->Z}

Key Idea: Worklist of Contexts

val $worklist : Set[Context]$
val $analyzing : Set[Context]$
val $results : Map[Context, Summary]$
val $callers : Map[Context, Set[Context]]$

function ANALYZEPROGRAM
    $initCtx \leftarrow$ GETCTX(main, **nil**, 0, $\top$)
    $worklist \leftarrow \{initCtx\}$
    $results[initCtx] \leftarrow Summary(\top, \bot)$
    while NOTEMPTY($worklist$) **do**
        $ctx \leftarrow$ REMOVE($worklist$)
        ANALYZE($ctx, results[ctx].input$)
    end while
end function

function ANALYZE(ctx, σ_{in})
    $\sigma_{out} \leftarrow results[ctx].output$
    ADD($analyzing, ctx$)
    $\sigma'_{out} \leftarrow$ INTRAPROCEDURAL(ctx, σ_{in})
    REMOVE($analyzing, ctx$)
    if $\sigma'_{out} \not\sqsubseteq \sigma_{out}$ **then**
        $results[ctx] \leftarrow Summary(\sigma_{in}, \sigma_{out} \sqcup \sigma'_{out})$
        for $c \in callers[ctx]$ **do**
            ADD($worklist, c$)
        end for
    end if
    return σ'_{out}
end function

```
function ResultsFor(ctx, σ_in)
    if ctx ∈ dom(results) then
        if σ_in ⊑ results[ctx].input then
            return results[ctx].output                          ▻ existing results are good
        else
            results[ctx].input ← results[ctx].input ⊔ σ_in    ▻ keep track of more general input
        end if
    else
        results[ctx] = Summary(σ_in, ⊥)                         ▻ initially optimisti
    end if
    if ctx ∈ analyzing then
        return results[ctx].output      ▻ ⊥ if it hasn't been analyzed yet; otherwis
    else
        return Analyze(ctx, results[ctx].input)
    end if
end function
```

```
function Flow([[n: x := f(y)]], ctx, σ_n)
    σ_in ← [formal(f) ↦ σ_n(y)]                 ▻
    calleeCtx ← GetCtx(f, ctx, n, σ_in)
    σ_out ← ResultsFor(calleeCtx, σ_in)
    Add(callers[calleeCtx], ctx)
    return σ_n[x ↦ σ_out[result]]
```

```
function Analyze(ctx, σ_in)
    σ_out ← results[ctx].output
    Add(analyzing, ctx)
    σ'_out ← Intraprocedural(ctx, σ_in)
    Remove(analyzing, ctx)
    if σ'_out ⋢ σ_out then
        results[ctx] ← Summary(σ_in, σ_out ⊔ σ'_out)
        for c ∈ callers[ctx] do
            Add(worklist, c)
        end for
    end if
    return σ'_out
end function
```

Extending WHILE3ADDR with Pointers

$$
\begin{array}{rcll}
I & ::= & \ldots & \\
  & | & p := \&x & \text{taking the address of a variable} \\
  & | & p := q & \text{copying a pointer from one variable to another} \\
  & | & *p := q & \text{assigning through a pointer} \\
  & | & p := *q & \text{dereferencing a pointer}
\end{array}
$$

Andersen's Analysis

$$\frac{}{[\![p := \&x]\!] \hookrightarrow l_x \in p} \textit{ address-of}$$

$$\frac{}{[\![p := q]\!] \hookrightarrow p \supseteq q} \textit{ copy}$$

$$\frac{}{[\![*p := q]\!] \hookrightarrow *p \supseteq q} \textit{ assign}$$

$$\frac{}{[\![p := *q]\!] \hookrightarrow p \supseteq *q} \textit{ dereference}$$

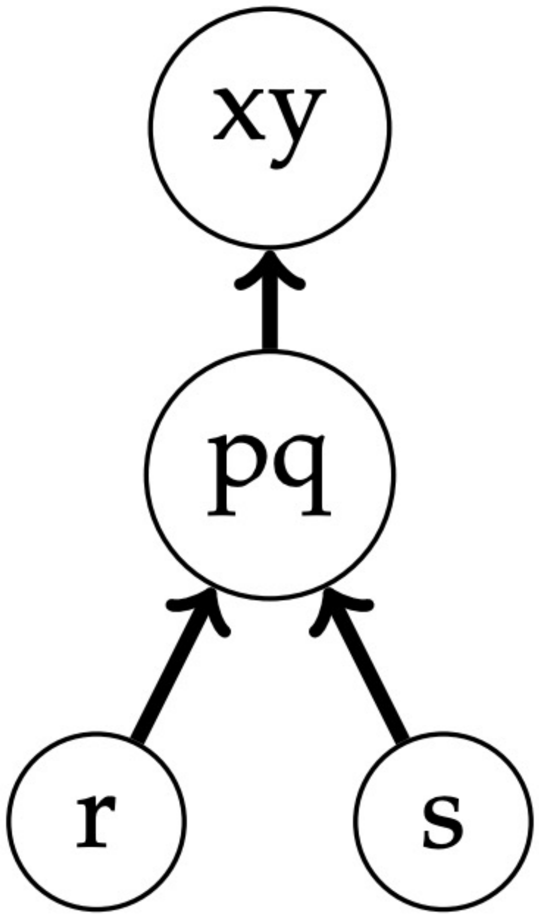
$$\frac{p \supseteq q \quad l_x \in q}{l_x \in p} \textit{ copy}$$

$$\frac{*p \supseteq q \quad l_r \in p \quad l_x \in q}{l_x \in r} \textit{ assign}$$

$$\frac{p \supseteq *q \quad l_r \in q \quad l_x \in r}{l_x \in p} \textit{ dereference}$$

Steensgaard's Analysis - Example

$$
\begin{aligned}
1:&\quad p := \&x \\
2:&\quad r := \&p \\
3:&\quad q := \&y \\
4:&\quad s := \&q \\
5:&\quad r := s
\end{aligned}
$$



Steensgaard's Analysis

$$\frac{}{[\![p := q]\!] \hookrightarrow join(*p, *q)}\ copy$$

$$\frac{}{[\![p := \&x]\!] \hookrightarrow join(*p, x)}\ address\text{-}of$$

$$\frac{}{[\![p := *q]\!] \hookrightarrow join(*p, **q)}\ dereference$$

$$\frac{}{[\![*p := q]\!] \hookrightarrow join(**p, *q)}\ assign$$

```
join(ℓ1,ℓ2)
    if (find(ℓ1) == find(ℓ2))
        return
    n1 ← *ℓ1
    n2 ← *ℓ2
    union(ℓ1,ℓ2)
    join(n1,n2)
```

Analyzing Functional Programming Languages

e	$\in$	$Expressions$	...or labelled terms
t	$\in$	$Term$	...or unlabelled expressions
l	$\in$	$\mathcal{L}$	labels

$$
\begin{array}{lcl}
e & ::= & t^l \\
t & ::= & \lambda x.e \\
 & | & x \\
 & | & (e_1)\ (e_2) \\
 & | & \text{let } x = e_1 \text{ in } e_2 \\
 & | & \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \\
 & | & n \mid e_1 + e_2 \mid \ldots
\end{array}
$$

Simple 0-CFA Example

$$((\lambda x.(x^a + 1^b)^c)^d (3)^e)^g$$

$(\sigma(x) \sqsubseteq \sigma(a))$	*var*
$(\{\lambda x.x + 1\} \sqsubseteq \sigma(d))$	*lambda*
$(\sigma(e) \sqsubseteq \sigma(x)) \wedge (\sigma(c) \sqsubseteq \sigma(g))$	*apply* *function-flow*
$(\alpha(3) \sqsubseteq \sigma(e))$	*const*
$(\alpha(1) \sqsubseteq \sigma(b))$	*const*
$(\sigma(a) +_\top \sigma(b) \sqsubseteq \sigma(c))$	*plus*

0-CFA with Constant Propagation

$$(((\lambda f.(f^a\ 3^b)^c)^e(\lambda x.(x^g + 1^h)^i)^j)^k$$

$Var \cup Lab$	L	by rule
e	$\lambda f.f\ 3$	lambda
j	$\lambda x.x+1$	lambda
f	$\lambda x.x+1$	apply
a	$\lambda x.x+1$	var
b	3	const
x	3	apply
g	3	var
h	1	const
i	4	add
c	4	apply
k	4	apply

m-CFA

$$\textbf{let } add = \lambda x.\ \lambda y.\ x + y$$
$$\textbf{let } add5 = (add\ 5)^{a5}$$
$$\textbf{let } add6 = (add\ 6)^{a6}$$
$$\textbf{let } main = (add5\ 2)^{m}$$

Var / Lab, δ	L	notes
add, $\bullet$	$(\lambda x.\ \lambda y.\ x + y,\ \bullet)$	
x, a5	5	
add5, $\bullet$	$(\lambda y.\ x + y,\ a5)$	
x, a6	6	
add6, $\bullet$	$(\lambda y.\ x + y,\ a6)$	
main, $\bullet$	7	

Hoare Triple

$$\{ P \} \; S \; \{ Q \}$$

- P is the precondition
- Q is the postcondition
- S is any statement (in WHILE, at least for our class)

- Semantics: if P holds in some state E and if $<S;\, E> \Downarrow E'$, then Q holds in E'
  - This is *partial correctness*: termination of S is not guaranteed
  - *Total correctness* additionally implies termination, and is written $[P]\; S\; [Q]$

Semantics of Hoare Triples

- A partial correctness assertion $\models \{P\}\, S\, \{Q\}$ is defined formally to mean:

$$\forall E.\forall E'.(E \models P \wedge \langle E, S\rangle \Downarrow E') \Rightarrow E' \models Q$$

- How would we define total correctness $[P]\, S\, [Q]$?

- This is a good formal definition—but it doesn't help us prove many assertions because we have to reason about all environments. How can we do better?

Derivation Rules for Hoare Logic

- Judgment form $\vdash \{P\}\ S\ \{Q\}$ means "we can prove the Hoare triple $\{P\}\ S\ \{Q\}$"

$$\frac{}{\vdash \{P\}\ \mathrm{skip}\ \{P\}}\ \textit{skip} \qquad \frac{}{\vdash \{[a/x]P\}\ x{:=}a\ \{P\}}\ \textit{assign}$$

$$\frac{\vdash \{P\}\ S_1\ \{P'\} \qquad \vdash \{P'\}\ S_2\ \{Q\}}{\vdash \{P\}\ S_1;\ S_2\ \{Q\}}\ \textit{seq} \qquad \frac{\vdash \{P \wedge b\} S_1 \{Q\} \qquad \vdash \{P \wedge \neg b\}\ S_2\ \{Q\}}{\vdash \{P\}\ \mathbf{if}\ b\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \{Q\}}\ \textit{if}$$

$$\frac{\vdash P' \Rightarrow P \qquad \vdash \{P\}\ S\ \{Q\} \qquad \vdash Q \Rightarrow Q'}{\vdash \{P'\}\ S\ \{Q'\}}\ \textit{consq}$$

Hoare Triples and Weakest Preconditions

- Theorem: {P} S {Q} holds if and only if $P \Rightarrow wp(S,Q)$
  - In other words, a Hoare Triple is still valid if the precondition is stronger than necessary, but not if it is too weak
  - Can use this to prove {P} S {Q} by computing wp(S,Q) and checking implication.

- Question: Could we state a similar theorem for a strongest postcondition function?
  - e.g. {P} S {Q} holds if and only if $sp(S,P) \Rightarrow Q$
  - A: Yes, but it's harder to compute (see text for why)

Proving loops correct

- First consider *partial correctness*
  - The loop may not terminate, but if it does, the postcondition will hold
- {P} while B do S {Q}
  - Find an invariant Inv such that:
    - $P \Rightarrow Inv$
      - The invariant is initially true
    - { Inv && B } S {Inv}
      - Each execution of the loop preserves the invariant
    - $(Inv \;\&\&\; \neg B) \Rightarrow Q$
      - The invariant and the loop exit condition imply the postcondition

What if we just went forwards?

$$\{P\}$$
$$x := e_1$$
$$x := e_2$$
$$\{Q\}$$

Generate "fresh" math variables for every mutable program variable

Proof Obligation:

$$\forall x_n : ([x_0/x]P \wedge x_1 = [x_0/x]e_1 \wedge x_2 = ([x_1/x]e_2)) \Rightarrow [x_2/x]Q$$

Dealing with conditional paths

$$\{true\}$$
$$\textbf{if}\ (x < 0):$$
$$\quad y := -x$$
$$\textbf{else}:$$
$$\quad y := x$$
$$\{y \geqslant 0\}$$

Dynamic Symbolic Execution:

$$\forall x_0, y_0 \in \mathbb{Z} : (x_0 < 0 \wedge y_0 = -x_0) \Rightarrow y_0 \geqslant 0$$

$$\forall x_0, y_0 \in \mathbb{Z} : (x_0 \geqslant 0 \wedge y_0 = x_0) \Rightarrow y_0 \geqslant 0$$

Static Symbolic Execution:

$$\forall x_0, y_0 \in \mathbb{Z} : ((x_0 < 0 \Rightarrow y_0 = -x_0) \vee (x_0 \geqslant 0 \Rightarrow y_0 = x_0)) \Rightarrow y_0 \geqslant 0$$

Symbolic Execution of Statements (DSE)

$$\frac{}{\langle g, \Sigma, \texttt{skip}\rangle \Downarrow \langle g, \Sigma\rangle}\ \textit{big-skip}$$

$$\frac{\langle g, \Sigma, s_1\rangle \Downarrow \langle g', \Sigma'\rangle \quad \langle g', \Sigma', s_2\rangle \Downarrow \langle g'', \Sigma''\rangle}{\langle g, \Sigma, s_1; s_2\rangle \Downarrow \langle g'', \Sigma''\rangle}\ \textit{big-seq}$$

$$\frac{\langle \Sigma, a\rangle \Downarrow a_s}{\langle g, \Sigma, x := a\rangle \Downarrow \langle g, \Sigma[x \mapsto a_s]\rangle}\ \textit{big-assign}$$

Symbolic Execution with Branching (DSE)

$$\frac{\langle \Sigma, b \rangle \Downarrow g' \quad g \wedge g' \texttt{SAT} \quad \langle g \wedge g', \Sigma, s_1 \rangle \Downarrow \langle g'', \Sigma' \rangle}{\langle g, \Sigma, \texttt{if}\ b\ \texttt{then}\ s_1\ \texttt{else}\ s_2, \rangle \Downarrow \langle g'', \Sigma' \rangle}\ \textit{big-iftrue}$$

$$\frac{\langle \Sigma, b \rangle \Downarrow g' \quad g \wedge \neg g' \texttt{SAT} \quad \langle g \wedge \neg g', \Sigma, s_2 \rangle \Downarrow \langle g'', \Sigma' \rangle}{\langle g, \Sigma, \texttt{if}\ b\ \texttt{then}\ s_1\ \texttt{else}\ s_2, \rangle \Downarrow \langle g'', \Sigma' \rangle}\ \textit{big-iffalse}$$

Symbolic Execution of Loops

Bounded exploration (*k*-limited)

$$\frac{k > 0 \quad \langle \Sigma, b\rangle \Downarrow g' \quad g \wedge g'\,\mathtt{SAT} \quad \langle g \wedge g', \Sigma, s; \mathbf{while_{k-1}}\ b\ \mathbf{do}\ s\rangle \Downarrow \langle g'', \Sigma'\rangle}{\langle g, \Sigma, \mathbf{while_k}\ b\ \mathbf{do}\ s, \rangle \Downarrow \langle g'', \Sigma'\rangle}\ \textit{big-whiletrue}$$

$$\frac{\langle \Sigma, b\rangle \Downarrow g' \quad g \wedge \neg g'\,\mathtt{SAT}}{\langle g, \Sigma, \mathbf{while_k}\ b\ \mathbf{do}\ s, \rangle \Downarrow \langle g \wedge \neg g', \Sigma\rangle}\ \textit{big-whilefalse}$$

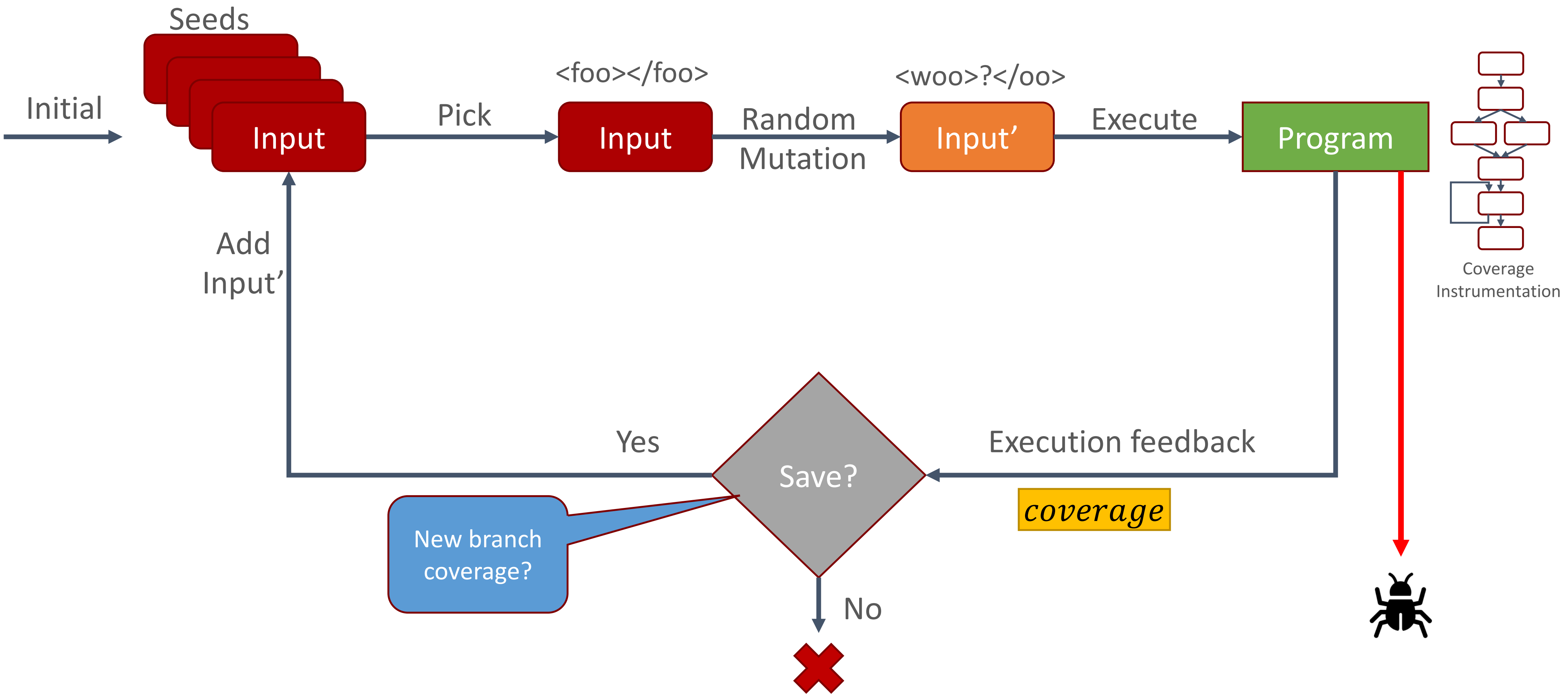
Concolic Execution

```
int double (int v) {
    return 2*v;
}

void bar(int x, int y) {
    z = double (y);
    if (z == x) {
        if (x > y+10) {
            ERROR;
        }
    }
}
```

1. Input: x=0, y=1
   - Path: (2*y != x)
   - Next: (2*y == x) :: **SAT**
2. Input: x=2, y=1
   - Path: (2*y == x) && (x <= y+10)
   - Next: (2*y == x) && (x > y+10) :: **SAT**
3. Input: x=22, y=11
   - Path: (2*y == x) && (x > y+10)
   - **Bug found!!**

Coverage-Guided Fuzzing with AFL



Satisfiability (SAT) solving

- Let's start by considering Boolean formulas: variables connected with $\wedge \vee \neg$
- First step: convert to conjuctive normal form (CNF)
  - A conjunction of disjunctions of (possibly negated) variables

$$(a \vee \neg b) \wedge (\neg a \vee c) \wedge (b \vee c)$$

- If formula is not in CNF, we transform it: use De Morgan's laws, the double negative law, and the distributive laws:

$$\begin{aligned}
\neg(P \vee Q) &\iff \neg P \wedge \neg Q \\
\neg(P \wedge Q) &\iff \neg P \vee \neg Q \\
\neg\neg P &\iff P \\
(P \wedge (Q \vee R)) &\iff ((P \wedge Q) \vee (P \wedge R)) \\
(P \vee (Q \wedge R)) &\iff ((P \vee Q) \wedge (P \vee R))
\end{aligned}$$

The Full DPLL Algorithm

```
function DPLL(φ)
    if φ = true then
        return true
    end if
    if φ contains a false clause then
        return false
    end if
    for all unit clauses l in φ do
        φ ← UNIT-PROPAGATE(l, φ)
    end for
    for all literals l occurring pure in φ do
        φ ← PURE-LITERAL-ASSIGN(l, φ)
    end for
    l ← CHOOSE-LITERAL(φ)
    return DPLL(φ ∧ l) ∨ DPLL(φ ∧ ¬l)
end function
```

Heuristic: Apply unit propagation first because it creates more units and pure literals. Pure literal assignment only removes entire clauses.

Try both assignments of the chosen literal. If we assume ∨ is short-circuiting, then this implements backtracking.

Satisfiability Modulo Theories

- Theory of uninterpreted functions

$$f(e1) = a$$
$$e2 = f(x)$$
$$e3 = f(y)$$
$$f(e4) = e5$$
$$x = y$$

- Theory of arithmetic

$$e1 = e2 - e3$$
$$e4 = 0$$
$$e5 = a + 2$$
$$x = y$$

- Congruence closure:

for all f, x, and y, if $x = y$ then $f(x) = f(y)$

Theories communicate using equalities

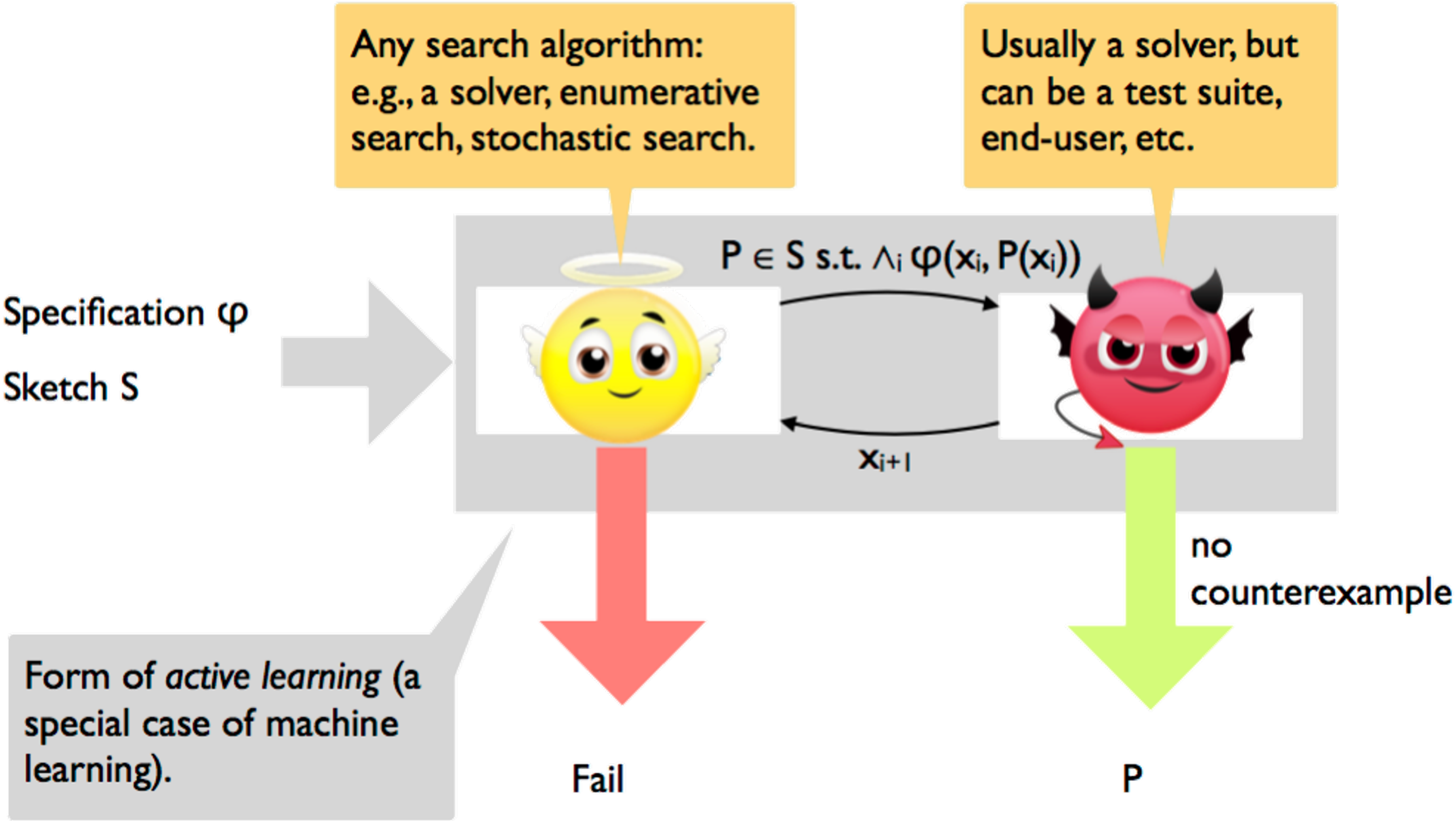
Program Synthesis Overview

- A mathematical characterization of program synthesis: prove that

$$\exists P \,.\, \forall x \,.\, \varphi(x, P(x))$$

- In constructive logic, the witness to the proof of this statement is a program P that satisfies property φ for all input values x
- What could the inferred program P be?
  - Historically, a protocol, interpreter, classifier, compression algorithm, scheduling policy, cache coherence policy, …
- How is property φ expressed?
  - Historically, as a formula, a reference implementation, input/output pairs, traces, demonstrations, a sketch, …

Overview of CEGIS



Oracle-guided component-based synthesis

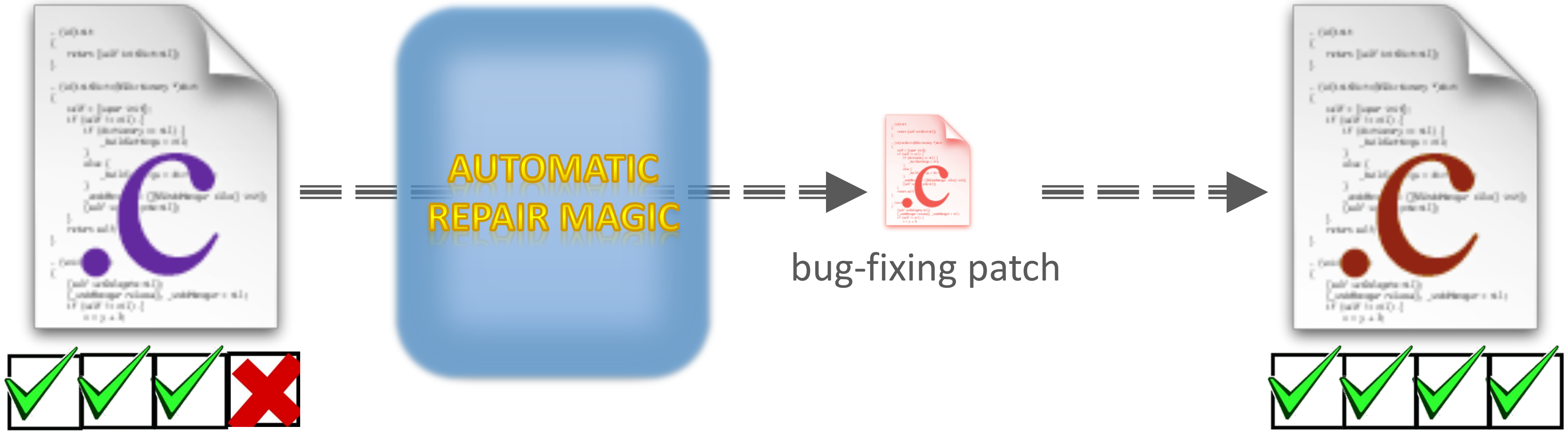
- Program input variable $\overrightarrow{Y}$
- Input to each component
- Output of each component
- Output of the program r

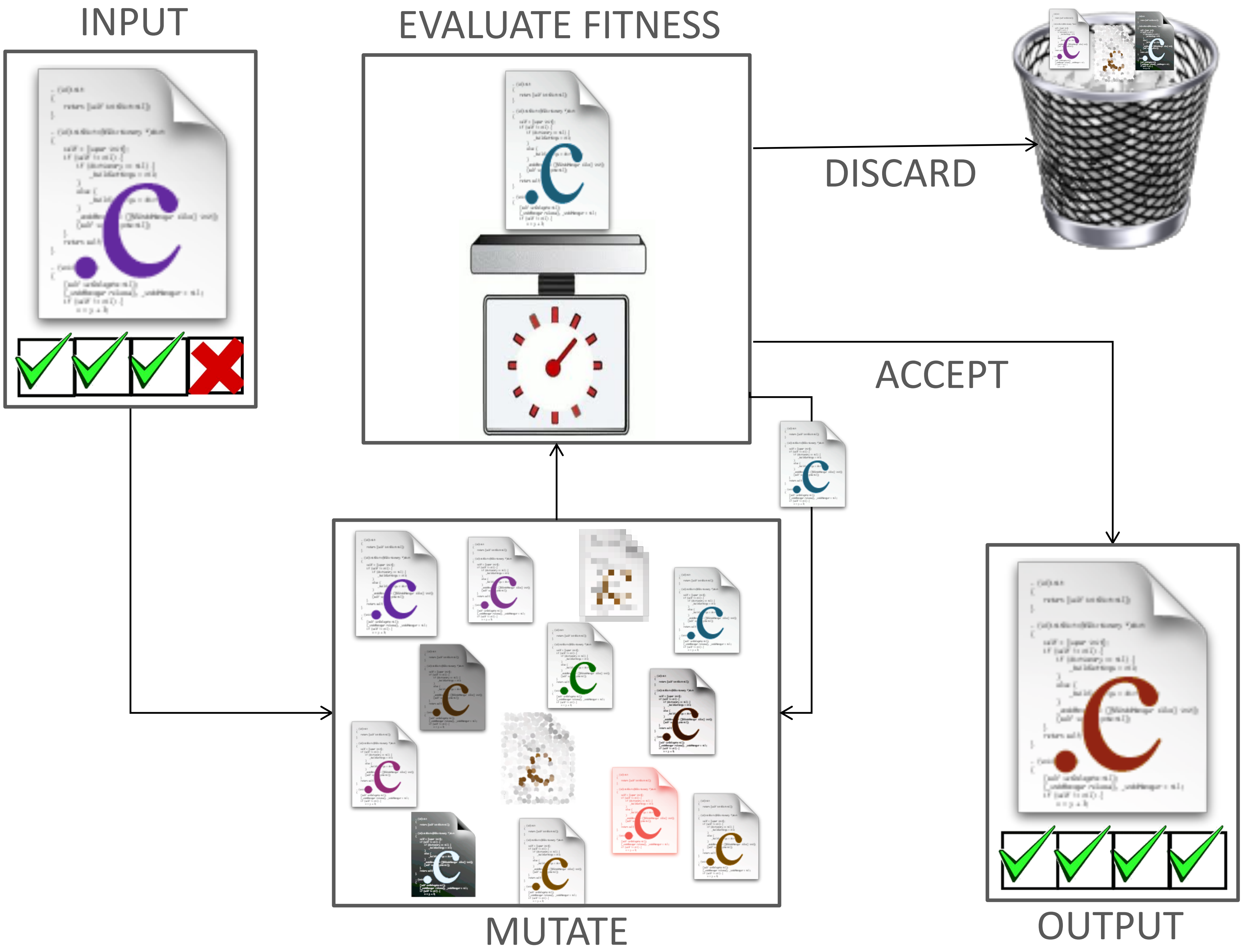
$$Q := \bigcup_{i=1}^{N} \overrightarrow{\chi}_i$$

$$R := \bigcup_{i=1}^{N} r_i$$

0	$z_0 := \texttt{input}^0$
1	$z_1 := \texttt{input}^1$
$\dots$	$\dots$
m	$z_m := \texttt{input}^m$
$m+1$	$z_{m+1} := f_?(z_?, \dots, z_?)$
$m+2$	$z_{m+2} := f_?(z_?, \dots, z_?)$
$\dots$	$\dots$
$m+N$	$z_{m+N} := f_?(z_?, \dots, z_?)$
$m+N+1$	$\texttt{return}\ z_?$

Automatic Program Repair




INPUT
EVALUATE FITNESS
DISCARD
ACCEPT
MUTATE
OUTPUT

```
1  int is_upward( int inhibit, int up_sep, int down_sep){
2        int bias;
3        if (inhibit)
4            bias = α;
5        else  bias = up_sep ;
6        if (bias > down_sep)
7             return 1;
8        else return 0;
9  }
```

inhibit	up_sep	down_sep	Observed output	Expected Output	**Result**
1	*11*	*110*	*0*	*1*	***fail***

Line 4

inhibit = 1, up_sep = 11, down_sep = 110
bias = **α**, PC = true

Line 7

inhibit = 1, up_sep = 11, down_sep = 110
bias = **α**, PC= **α** > 110

~~Line 8~~

~~inhibit = 1, up_sep = 11, [illegible]ep = 110~~
~~bias = **α**, PC= [illegible] 110~~

Dynamic analysis

- Observe program behavior during *execution* on *one or more inputs*.
- Examples:
  - Code coverage (→ Greybox fuzzing, fault localization)
  - Performance Profiling
    - Code profiling, memory profiling, algorithmic profiling
  - Invariant Generation
  - Concolic Execution
  - Data structure analysis
  - Concurrency analysis: Race detection
  - Concurrency analysis: Deadlock detection
  - Taint Analysis (→ Security & Privacy)
  - … (many many more)

Infer Likely Invariants

Program: (input= N >0)

```
i := 0
while i != N:
  i := i + 1
```

Loop Invariants to Evaluate

- i = 0
- i < 0
- i <= 0
- i > 0
- i >= 0
- N = 0
- N < 0
- N <= 0
- N >= 0
- N > 0
- i == N
- i < N
- i <= N
- i > N
- i >= N

Collecting execution info

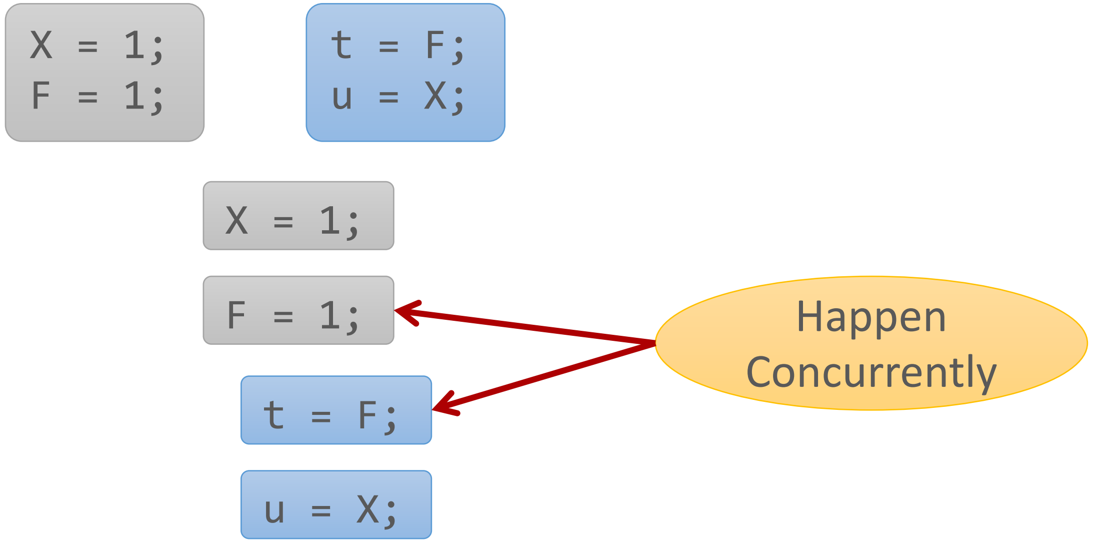
- What to collect? *Only what's necessary*
- Key idea (again): *Abstraction*
- Examples:
    - Code coverage → Log branches
    - Profiling → Log loops, function calls, allocations, frees, etc.
    - Invariant generation → Log predicates over vars in scope
    - Concolic execution → Track symbolic values; log branch constraints
    - Race detection → Track locks, vector clocks; log accesses

Stack Machine Bytecode

Instruction (at <label>)	Stack (before → after)
Push <const>	… → … <const>
Load <var>	… → … E(var)
Store <var>	… val → … // E[var ↦ val]
Dup	… val → … val val
Add	… val_1 val_2 → … (val_1+val_2)
Invoke <func> <nargs>	… val_1 val_2 … val_{nargs} → … result
Jump <label’>	… → … // PC = label’
Jump-if-zero <label’>	… val → … // PC = val ? PC+1 : label’

Data Races

- A data race is a pair of conflicting accesses that happen concurrently



Data Races Can Break Sequentially Consistent Semantics

Init: X = Y = 0;

```
L1: t = X*5;
L2: u = Y;
L3: v = X*5;
```

```
M1: X = 1;
M2: Y = 1;
```

u == 1 ➔ v == 5

Data Race

Init: X = Y = 0;

```
L1: t = X*5;
L2: u = Y;
L3: v = t;
```

```
M1: X = 1;
M2: Y = 1;
```

possibly u == 1 && v == 0

Lockset Algorithm Overview

- Checks a sufficient condition for data-race-freedom
- Consistent locking discipline
  - Every data structure is protected by a single lock
  - All accesses to the data structure made while holding the lock
- Example:

```
// Remove a received packet
AcquireLock( RecvQueueLk );
pkt = RecvQueue.RemoveTop();
ReleaseLock( RecvQueueLk );

… // process pkt

// Insert into processed
AcquireLock( ProcQueueLk );
ProcQueue.Insert(pkt);
ReleaseLock( ProcQueueLk );
```

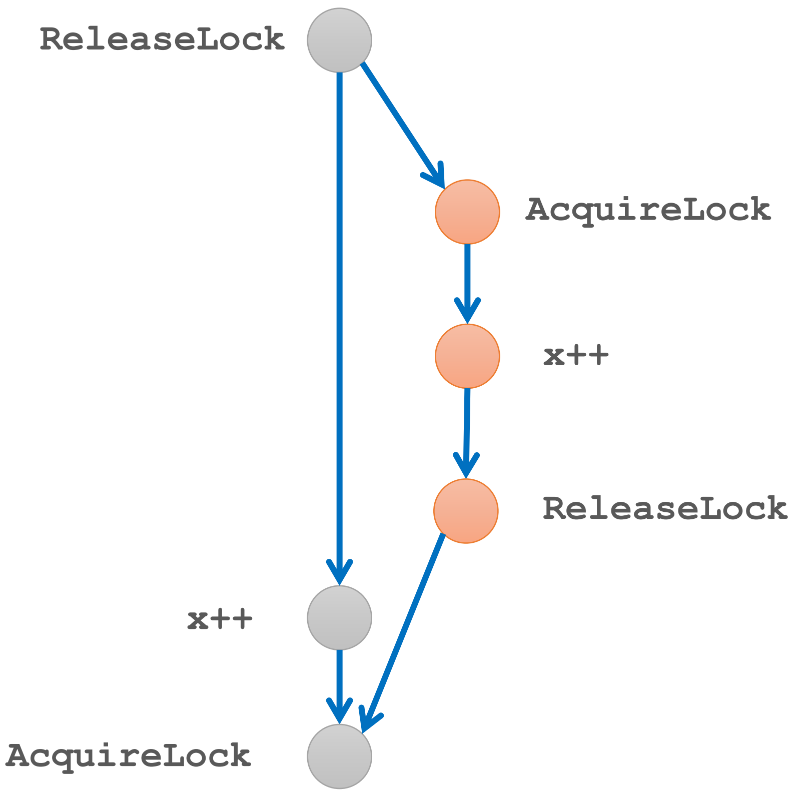
RecvQueue is consistently protected by RecvQueueLk

ProcQueue is consistently protected by ProcQueueLk

Happens-Before Relation And Data Races

- If all conflicting accesses are ordered by happens-before
- → data-race-free execution
- → All linearizations of partial-order are valid program executions

- If there exists conflicting accesses not ordered
- → a data race



Vector Clocks

VC_A	VC_B	L_m	W_x	R_x
4, 1	2, 8	2, 1	3, 0	0, 1

`x = 0`

Write-Write Check: $W_x \sqsubseteq VC_A$?

$[3, 0] \sqsubseteq [4, 1]$? Yes

Read-Write Check: $R_x \sqsubseteq VC_A$?

$[0, 1] \sqsubseteq [4, 1]$? Yes

O(n) time

Static vs Dynamic Analysis

- Over-approximation vs Under-approximation
- When is one better than other? Tradeoffs!
  - Soundness/Completeness
    - Static analysis often “sound” for over-approximate reasoning (e.g. verification)
    - Dynamic Analysis can be ”sound” for under-approximate reasoning (e.g. hot spots or bugs).
    - Neither technique is complete in general.
  - Scalability
    - Static analysis often scales super-linearly with program size
    - Dynamic analysis *tries* to scale linearly with execution length
  - Feasibility
    - Static analysis may be impossible with incomplete information (e.g. dynamically loaded code, dependency injection, multi-language code, hardware interaction)
    - Dynamic analysis is only useful when appropriate program inputs are available

Course Evaluations – **cmu.smartevals.com**

17-355 (Undergrad)

17-665 (Masters)

17-819 (PhD)

Claim participation points on Canvas after filling out eval: "Lecture 25 Quiz"

Next Steps – Course Project

- Checkpoint due tonight (April 28)
- Recitation and OH reserved for project discussions
- Project Presentations (May 9)
  - In-person 1-4pm at GHC 4307
  - Bring your laptops and adapters, if any
  - 6 min talks (firm time limit) + ~2min Q&A
  - Email me in advance if you need to Zoom in (talk must be recorded)
- Project Report due May 9 at midnight