Lecture 3: Data-Flow Analysis

17-355/17-655/17-819: Program Analysis

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Data-Flow Analysis

Computes universal properties about program state at specific program points. (e.g. will *x* be zero at line 7?)

- About program state
  - About data store (e.g. variables, heap memory)
  - Not about control (e.g. termination, performance)
- At program points
  - Statically identifiable (e.g. line 7, or when foo() calls bar())
  - Not dynamically computed (E.g. when x is 12 or when foo() is invoked 12 times)
- Universal
  - Reasons about all possible executions (always/never/maybe)
  - Not about specific program paths (see: symbolic execution, testing)

Abstraction

$$\sigma \in Var \to L$$

$$\alpha : \mathbb{Z} \to L$$

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Zero Analysis

$$L = \{Z, N, \top\}$$

$$\alpha_Z(0) = Z$$
$$\alpha_Z(n) = N \text{where } n \neq 0$$

Flow Functions for Zero Analysis

A flow function maps values from σ to σ

$f[\![I]\!]$ -- flow across instruction I (think: “abstract semantics”)

$$f_Z[\![x := 0]\!](\sigma) =$$

$$f_Z[\![x := n]\!](\sigma) =$$

$$f_Z[\![x := y]\!](\sigma) =$$

$$f_Z[\![x := y \ op \ z]\!](\sigma) =$$

$$f_Z[\![\text{goto } n]\!](\sigma) =$$

$$f_Z[\![\text{if } x = 0 \text{ goto } n]\!](\sigma) =$$

Flow Functions for Zero Analysis

A flow function maps values from σ to σ

$f[\![I]\!]$ -- flow across instruction I (think: “abstract semantics”)

$$
\begin{aligned}
f_Z[\![x := 0]\!](\sigma) &= \sigma[x \mapsto Z] \\
f_Z[\![x := n]\!](\sigma) &= \sigma[x \mapsto N] \ \text{ where } n \neq 0 \\
f_Z[\![x := y]\!](\sigma) &= \sigma[x \mapsto \sigma(y)] \\
f_Z[\![x := y \ op \ z]\!](\sigma) &= \sigma[x \mapsto \top] \\
f_Z[\![\text{goto } n]\!](\sigma) &= \sigma \\
f_Z[\![\text{if } x = 0 \text{ goto } n]\!](\sigma) &= \sigma
\end{aligned}
$$

Flow Functions for Zero Analysis

Specializing for Precision

$$
\begin{aligned}
f_Z[\![x := y - y]\!](\sigma) &= \\
f_Z[\![x := y + z]\!](\sigma) &=
\end{aligned}
$$

Flow Functions for Zero Analysis

Specializing for Precision

$$
\begin{aligned}
f_Z[\![x := y - y]\!](\sigma) &= \sigma[x \mapsto Z] \\
f_Z[\![x := y + z]\!](\sigma) &= \sigma[x \mapsto \sigma(y)] \quad \text{where } \sigma(z) = Z
\end{aligned}
$$

Exercise 1: Define another flow function for some arithmetic instruction and certain conditions where you can also provide a more precise result than ⊤

Flow Functions for Zero Analysis

Specializing for Precision

$$\begin{aligned} f_Z[\![\text{if } x = 0 \text{ goto } n]\!]_T(\sigma) &= \\ f_Z[\![\text{if } x = 0 \text{ goto } n]\!]_F(\sigma) &= \end{aligned}$$

Flow Functions for Zero Analysis

Specializing for Precision

$$
\begin{aligned}
f_Z[\![\text{if } x = 0 \text{ goto } n]\!]_T(\sigma) &= \sigma[x \mapsto Z] \\
f_Z[\![\text{if } x = 0 \text{ goto } n]\!]_F(\sigma) &= \sigma[x \mapsto N]
\end{aligned}
$$

Exercise 2: Define a flow function for a conditional branch testing whether a variable x < 0

Control-flow Graphs

$$
\begin{array}{ll}
1: & \text{if } x = 0 \text{ goto } 4 \\
2: & y := 0 \\
3: & \text{goto } 6 \\
4: & y := 1 \\
5: & x := 1 \\
6: & z := y
\end{array}
$$

$\boxed{1\text{: if } x = 0 \text{ goto } 4}$

$\boxed{2\text{: } y \coloneqq 0}$

$\boxed{3\text{: goto } 6}$

$\boxed{4\text{: } y \coloneqq 1}$

$\boxed{5\text{: } x \coloneqq 1}$

$\boxed{6\text{: } z \coloneqq y}$

Nodes = Statements
Edges = (s1, s2) is an edge iff s1 and s2 can be executed consecutively aka ”control flow”

Control-flow Graphs

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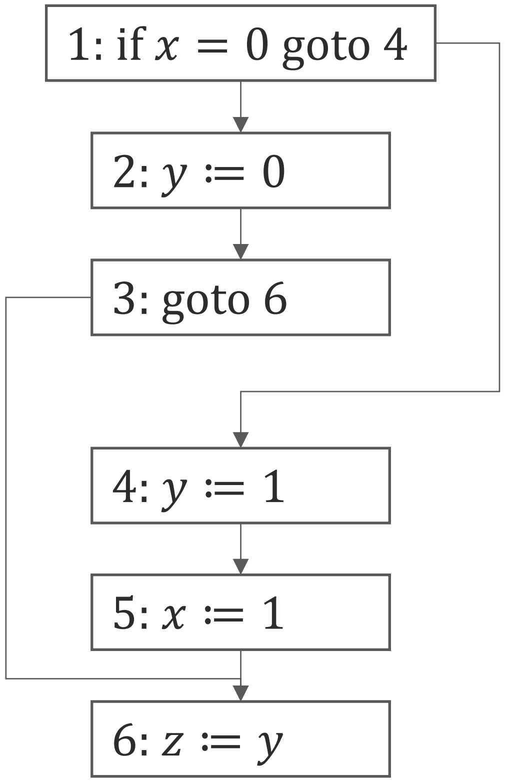


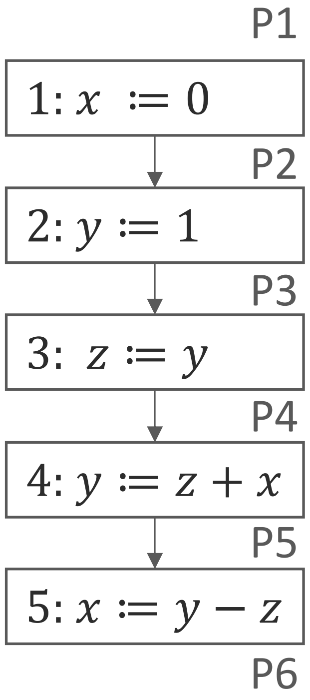
Nodes = Statements
Edges = (s1, s2) is an edge iff s1 and s2 can be executed consecutively aka ”control flow”

Common properties of CFGs:

- Weakly connected
- Only one entry node
- Only one exit (terminal) node

Example of Zero Analysis: Straightline Code

$$
\begin{aligned}
1:&\quad x := 0\\
2:&\quad y := 1\\
3:&\quad z := y\\
4:&\quad y := z + x\\
5:&\quad x := y - z
\end{aligned}
$$



	x	y	z
P1			
P2			
P3			
P4			
P5			
P6			

Example of Zero Analysis: Straightline Code

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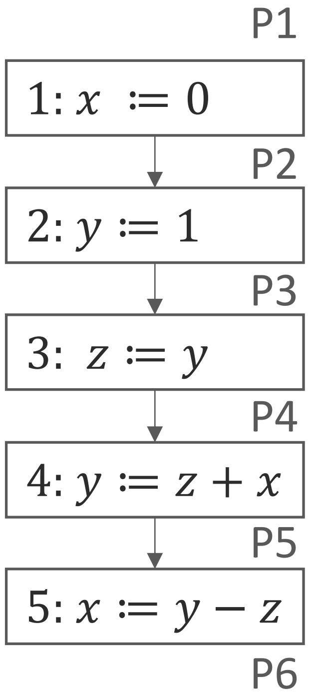
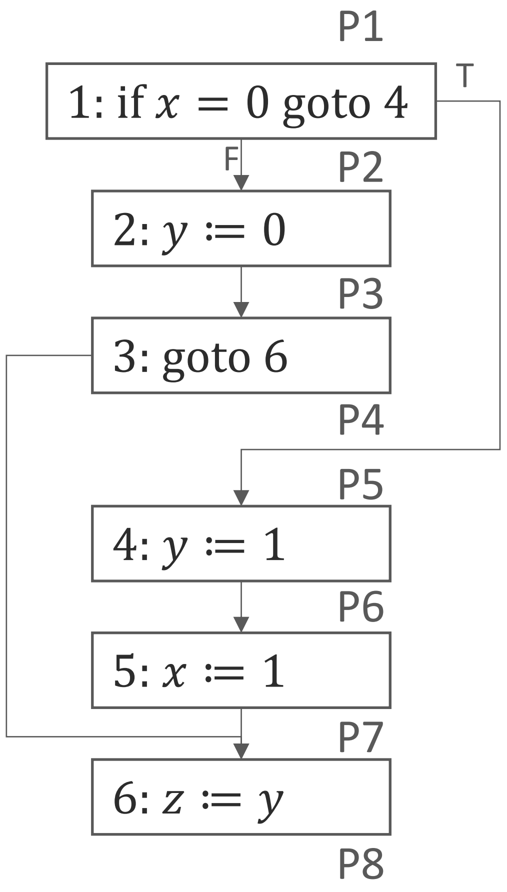


	x	y	z
P1	?	?	?
P2	Z	?	?
P3	Z	N	?
P4	Z	N	N
P5	Z	N	N
P6	⊤	N	N

Example of Zero Analysis: Branching Code

$$
\begin{array}{ll}
1: & \text{if } x = 0 \text{ goto } 4 \\
2: & y := 0 \\
3: & \text{goto } 6 \\
4: & y := 1 \\
5: & x := 1 \\
6: & z := y
\end{array}
$$



	x	y	z
P1			
P2			
P3			
P4			
P5			
P6			
P7			
P8			

Example of Zero Analysis: Branching Code

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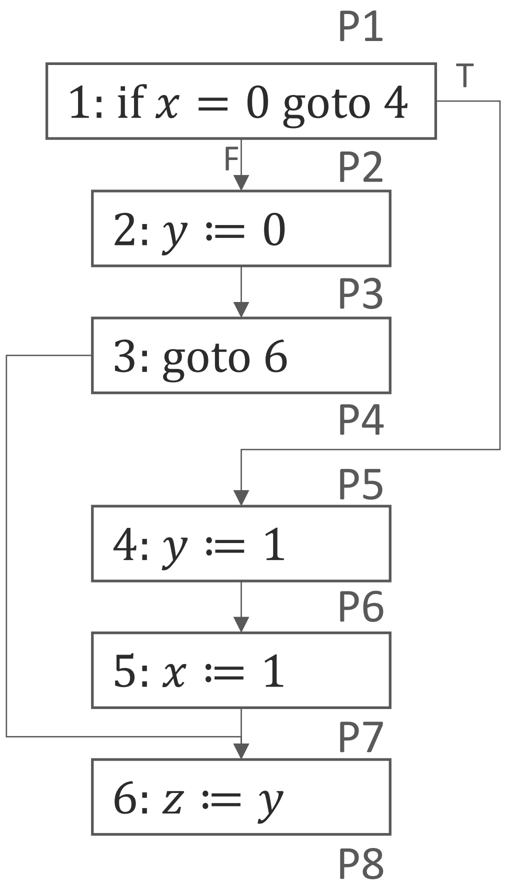


	x	y	z
P1	?	?	?
P2	Z_T, N_F	?	?
P3	N	Z	?
P4	N	Z	?
P5	Z	?	?
P6	Z	N	?
P7	N	$\top$	?
P8	N	$\top$	$\top$

Partial Order & Join on set L

$l_1 \sqsubseteq l_2$: l_1 is at least as precise as l_2

reflexive: $\forall l : l \sqsubseteq l$
transitive: $\forall l_1, l_2, l_3 : l_1 \sqsubseteq l_2 \wedge l_2 \sqsubseteq l_3 \Rightarrow l_1 \sqsubseteq l_3$
anti-symmetric: $\forall l_1, l_2 : l_1 \sqsubseteq l_2 \wedge l_2 \sqsubseteq l_1 \Rightarrow l_1 = l_2$

$l_1 \sqcup l_2$: **join** or *least-upper-bound*… "most precise generalization"

L is a *join-semilattice* iff: $l_1 \sqcup l_2$ always exists and is unique $\forall l_1, l_2 \in L$

$\top$ ("top") is the maximal element

Lattice for Zero Analysis

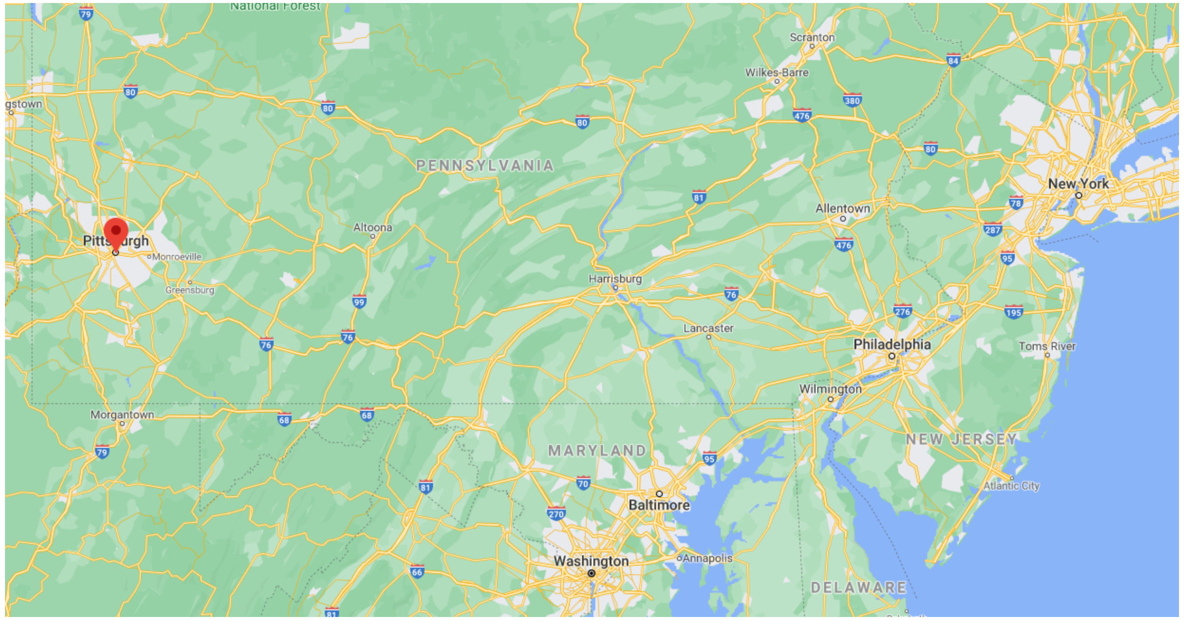
What would this look like?

Data-Flow Analysis

- a lattice $(L, \sqsubseteq)$
- an abstraction function α
- a flow function f
- initial dataflow analysis assumptions, σ_0

Random Facts #1

“You are here” maps don’t lie



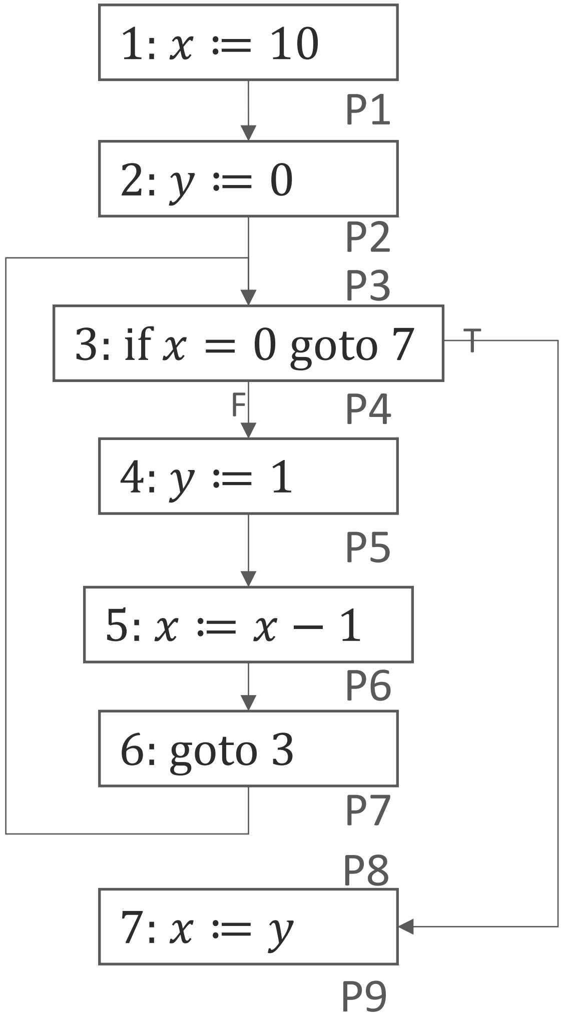
What mathematical concept is common to both these facts?

Python 3.8:

```
exec(s:='print("exec(s:=%r)"%s)')
```

Example of Zero Analysis: Looping Code

$$
\begin{array}{ll}
1: & x := 10 \\
2: & y := 0 \\
3: & \text{if } x = 0 \text{ goto } 7 \\
4: & y := 1 \\
5: & x := x - 1 \\
6: & \text{goto } 3 \\
7: & x := y
\end{array}
$$



	x	y
P0		
P1		
P2		
P3		
P4		
P5		
P6		
P7		
P8		
P9		

Example of Zero Analysis: Looping Code

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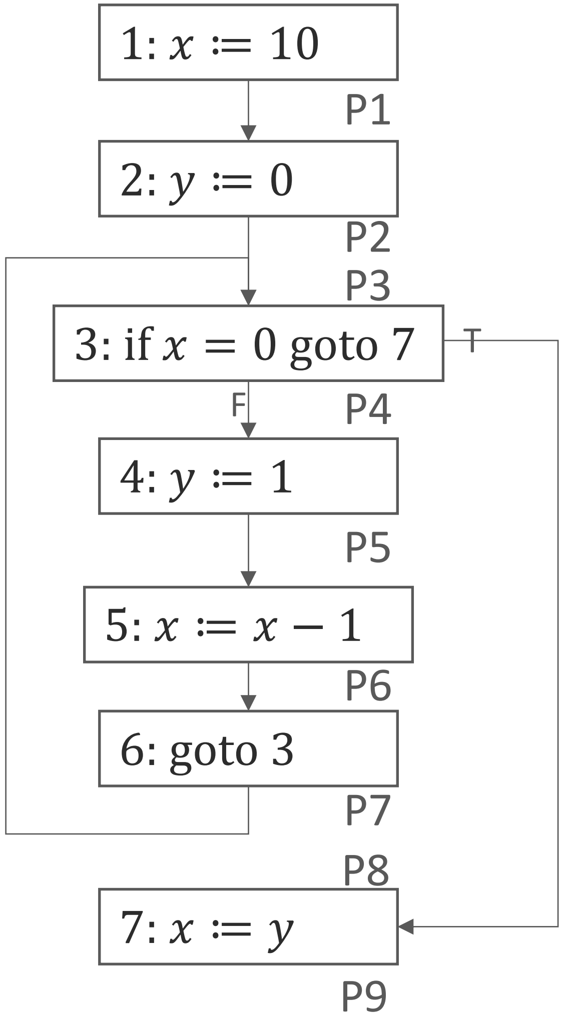

	x	y	
P0	$\top$	$\top$	
P1	N	$\top$	
P2	N	Z	
P3	N	Z	*first time through...*
P4	N_F	Z	
P5	N	N	
P6	$\top$	N	
P7	$\top$	N	
P8	Z_t	N	*first time through...*
P9	N	N	*first time through...*

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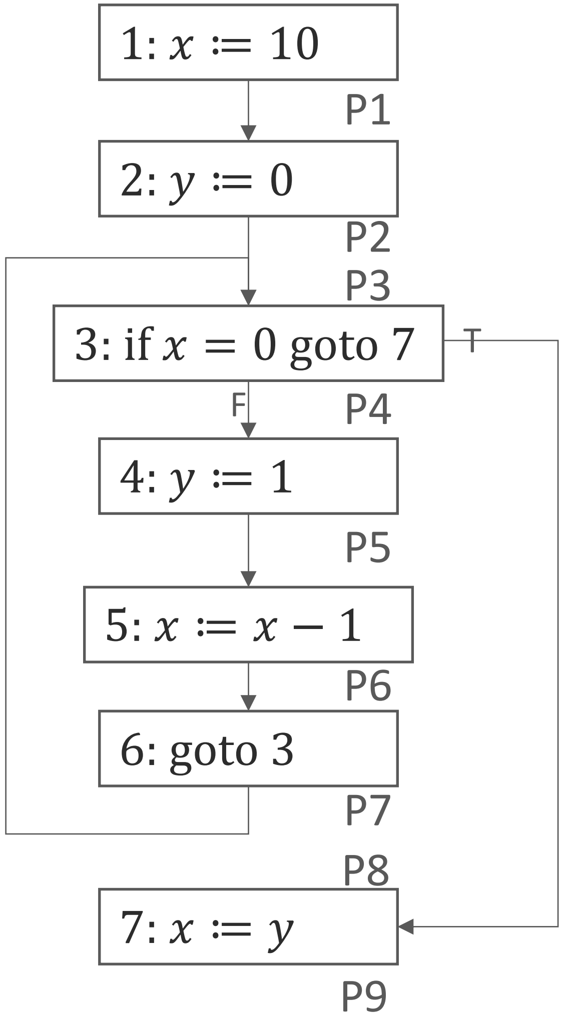

	x	y	
P0	$\top$	$\top$	
P1	N	$\top$	
P2	N	Z	
P3	$\top$	$\top$	*join*
P4	N_F	$\top$	*updated*
P5	N	N	*already at fixed point*
P6	$\top$	N	*already at fixed point*
P7	$\top$	N	*already at fixed point*
P8	Z_T	$\top$	*updated*
P9	$\top$	$\top$	*updated*