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These exercises prove properties of parity analysis. Assume the following:

- A lattice $(L, \sqsubseteq)$ where $L = \{\top, O, V, \bot\}$ and $\bot \sqsubseteq \{O, V\} \sqsubseteq \top$, $O \sqcup V = \top$
- An abstraction function $\alpha : \mathbb{Z} \mapsto L$, defined as follows:

$$\alpha(n) = \begin{cases} V \text{ when } n \text{ is an even integer } (n \in \{2k : k \in \mathbb{Z}\}) \\ O \text{ when } n \text{ is an odd integer } (n \in \{2k+1 : k \in \mathbb{Z}\}) \end{cases}$$

- a flow function f_P
- initial dataflow analysis assumptions σ_0, in this case σ_0 maps all variables' initial states to $\top$.

1. Disprove the local soundness of the incorrect flow function $f_P[\![a := b]\!](\sigma) = \sigma[a \mapsto O]$

For the next questions, use the following flow (correct) flow function for parity analysis:

$$f_P[\![a := b * c]\!](\sigma) = \begin{cases} \sigma[a \mapsto \bot] & \text{if } \sigma(b) = \bot \vee \sigma(c) = \bot \\ \sigma[a \mapsto O] & \text{if } \sigma(b) = O \wedge \sigma(c) = O \\ \sigma[a \mapsto V] & \text{if } (\sigma(b) = V \wedge \sigma(c) \neq \bot) \vee (\sigma(b) \neq \bot \wedge \sigma(c) = V) \\ \sigma[a \mapsto \top] & \text{if } (\sigma(b) = \top \wedge \sigma(c) \notin \{V, \bot\}) \vee (\sigma(b) \notin \{V, \bot\} \wedge \sigma(c) = \top) \end{cases}$$

2. Prove the monotonicity of $f_P[\![a := b * c]\!](\sigma)$ for the case $(\sigma(b) = V \wedge \sigma(c) \neq \bot) \vee (\sigma(b) \neq \bot \wedge \sigma(c) = V)$

3. Prove the local soundness of $f_P[\![a := b * c]\!](\sigma)$