Name and Andrew ID: _____

1. Use the big-step operational semantics rules for the WHILE language to write a well-formed derivation with $\langle y := 3; \texttt{if } y > 1 \texttt{ then } z := y \texttt{ else } z := 2, E\rangle \Downarrow E[y \mapsto 3; z \mapsto 3]$ as its conclusion. Make sure to indicate which rule you used to prove each premise or conclusion.

2. For homework 2, you will be partially proving that if a statement terminates, then the big- and small-step semantics for WHILE will obtain equivalent results; i.e.,

$$\forall S \in \texttt{Stmt}. \forall E, E' \in \texttt{Var} \mapsto \mathbb{Z}. \langle S, E\rangle \rightarrow^* \langle \texttt{skip}, E'\rangle \iff \langle S, E\rangle \Downarrow E'$$

   You will prove this by induction on the structure of derivations for each direction of $\iff$.

   For your homework proof, you are only required to show

   - The base case(s).
   - The inductive case for `let` using the semantics developed in question 1 of the homework.
   - Two more representative inductive cases.

You may assume that this property holds for arithmetic and boolean expressions, i.e., you may assume the following hold:

$$\forall a \in \texttt{AExp}.\forall n \in \mathbb{Z}.\langle a, E\rangle \rightarrow_a^* n \iff \langle a, E\rangle \Downarrow_a n \tag{1}$$

$$\forall P \in \texttt{BExp}.\forall b \in \{\texttt{true}, \texttt{false}\}.\langle P, E\rangle \rightarrow_b^* b \iff \langle P, E\rangle \Downarrow_b b \tag{2}$$

You may also assume the small-step if congruence of $\langle S, E\rangle \rightarrow^* \langle S', E'\rangle$:

$$\frac{\langle P, E\rangle \rightarrow_b^* P'}{\langle \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2, E\rangle \rightarrow^* \langle \texttt{if } P' \texttt{ then } S_1 \texttt{ else } S_2, E\rangle} \tag{3}$$

For this exercise, you will prove the following representative inductive case:

$$\forall S \in \texttt{Stmt}.\forall E, E' \in \texttt{Var} \mapsto \mathbb{Z}.\langle \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2, E\rangle \Downarrow E' \iff \langle \texttt{if } P \texttt{ then } S_1 \texttt{ else } S_2, E\rangle \rightarrow^* \langle \texttt{skip}, E'\rangle$$