

Software Model Checking and Counter-example Guided Abstraction Refinement

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Motivation: How should we analyze this?

```
2:  do {
      lock();
      old = new;
3:    if (*){
4:      unlock();
      new++;
    }
5:  } while (new != old);
6:  unlock();
   return;
```

- * means something we can't analyze (user input, random value)
- Line 5: the lock is held if and only if `old = new`

Motivation: How should we analyze this?

```
Example() {
1:  if (*){
7:      do {
           got_lock = 0;
8:           if (*){
9:               lock();
               got_lock++;
           }
10:          if (got_lock){
11:              unlock();
           }
12:      } while (*)
}
```

- * means something we can't analyze (user input, random value)
- Line 10: the lock is held if and only if got_lock = 1

Tradeoffs...

```
Example() {  
1:  if (*) {  
7:    do {  
        got_lock = 0;  
8:        if (*) {  
9:            lock();  
            got_lock++;  
        }  
10:       if (got_lock) {  
11:           unlock();  
        }  
12:    } while (*)  
}
```

```
2:  do {  
        lock();  
        old = new;  
3:        if (*) {  
4:            unlock();  
            new++;  
        }  
5:    } while (new != old);  
6:    unlock();  
    return;
```

Symbolic execution shows need to eliminate infeasible paths, see lock/unlock on correlated branches (more complicated logic!).

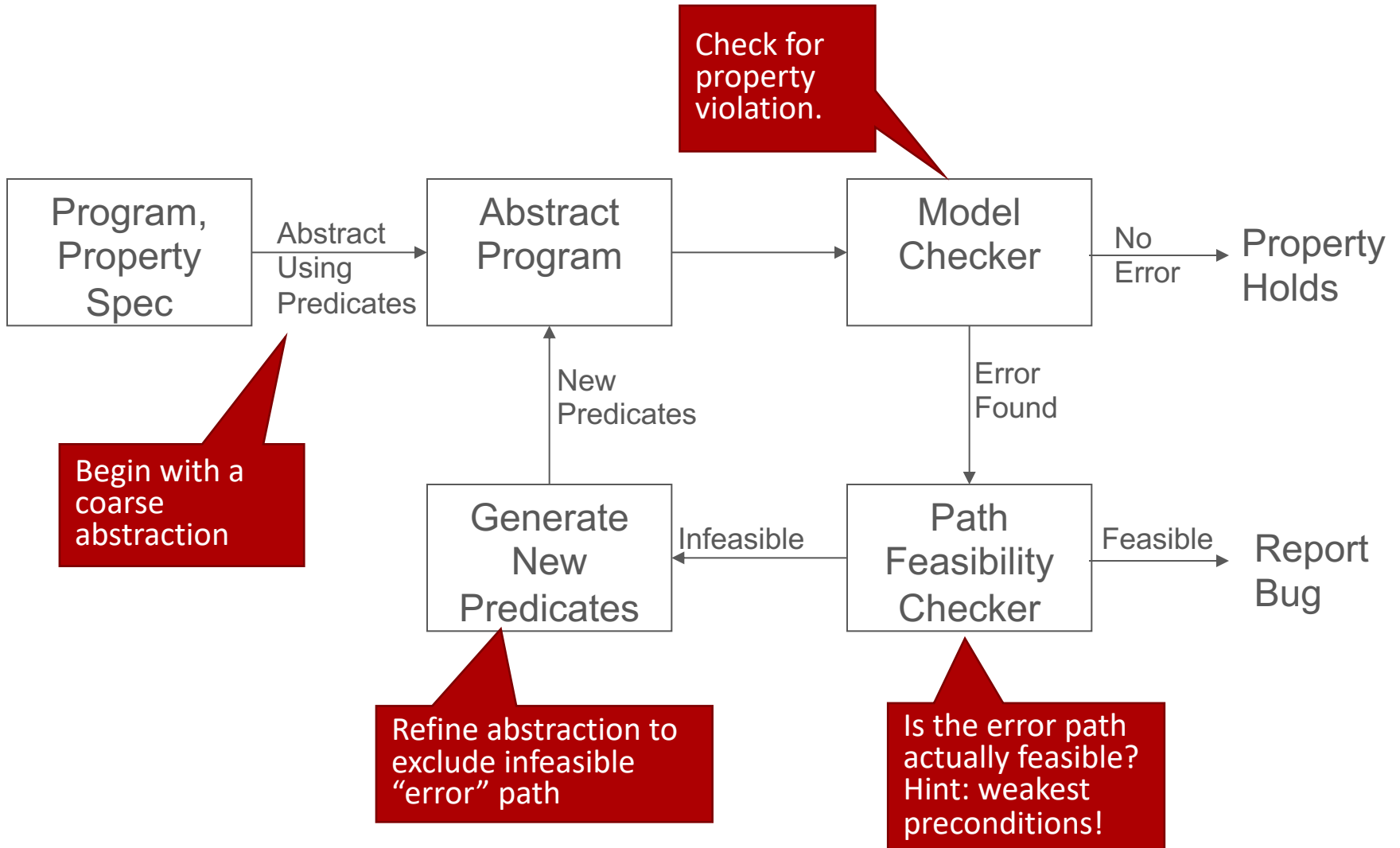
Dataflow analysis requires fixed abstractions, e.g., zero/non-zero, locked/unlocked

Explicit-state Model Checking needs programs to be represented as a finite state model...state explosion??

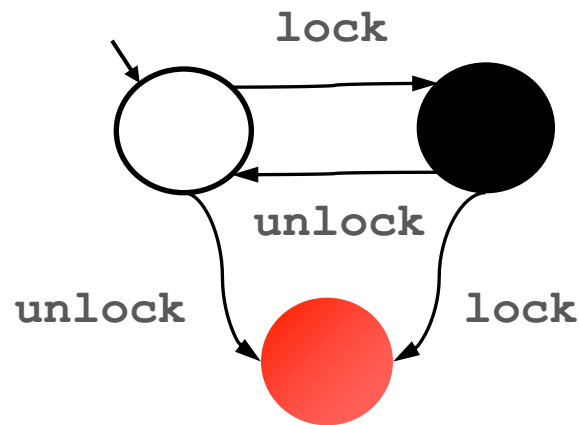
Enter: Abstraction Refinement

- Can we get both soundness and the precision to eliminate infeasible paths?
 - In general: of course not! That's undecidable.
 - But in many situations we can solve it with *abstraction refinement*.
- ...*what will we lose?*
 - Answer: Termination guarantees. OH WELL.

CEGAR: Counterexample Guided Abstraction Refinement



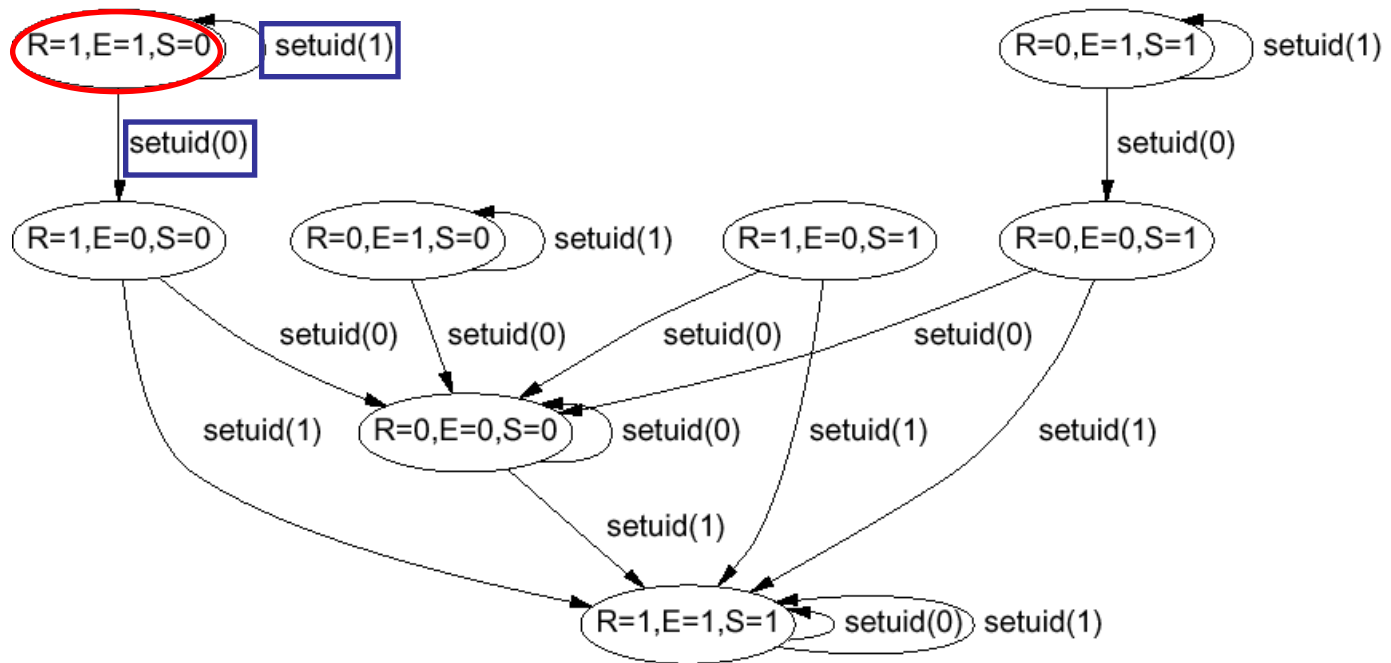
Property 1: Double Locking



*“An attempt to re-acquire an acquired lock or release a released lock will cause a **deadlock**.”*

Calls to **lock** and **unlock** must **alternate**.

Property 2: Drop Root Privilege

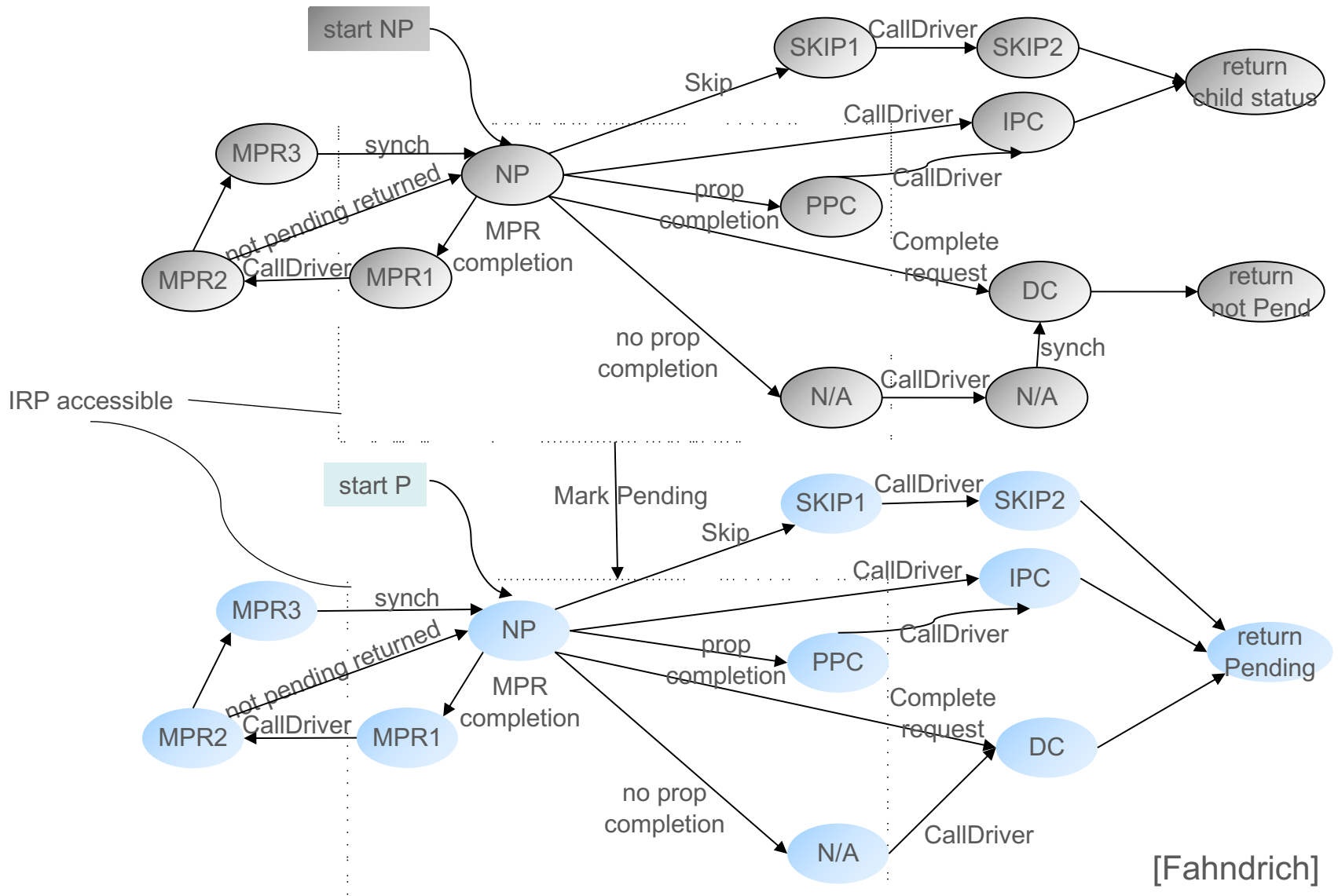


[Chen-Dean-Wagner '02]

“User applications must not run with root privilege”

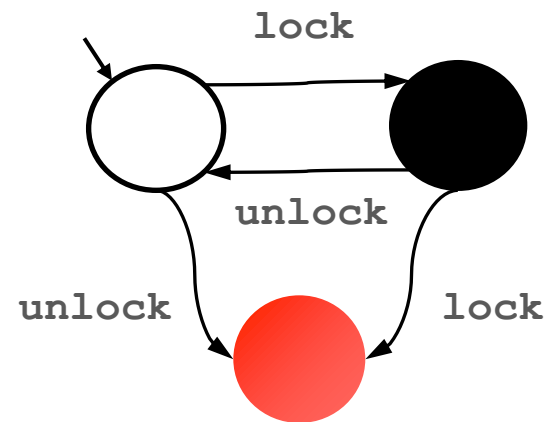
When **execv** is called, must have **suid ≠ 0**

Property 3 : IRP Handler



Example SLAM Input

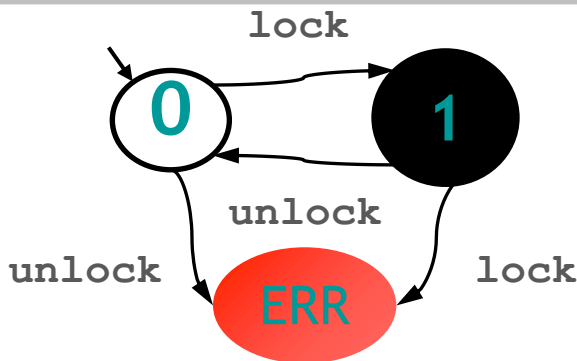
```
Example ( ) {  
1: do{  
    lock ();  
    old = new;  
    q = q->next;  
2:   if (q != NULL){  
3:     q->data = new;  
     unlock ();  
     new ++;  
    }  
4: } while(new != old);  
5:  unlock ();  
    return;  
}
```



Incorporating Specs

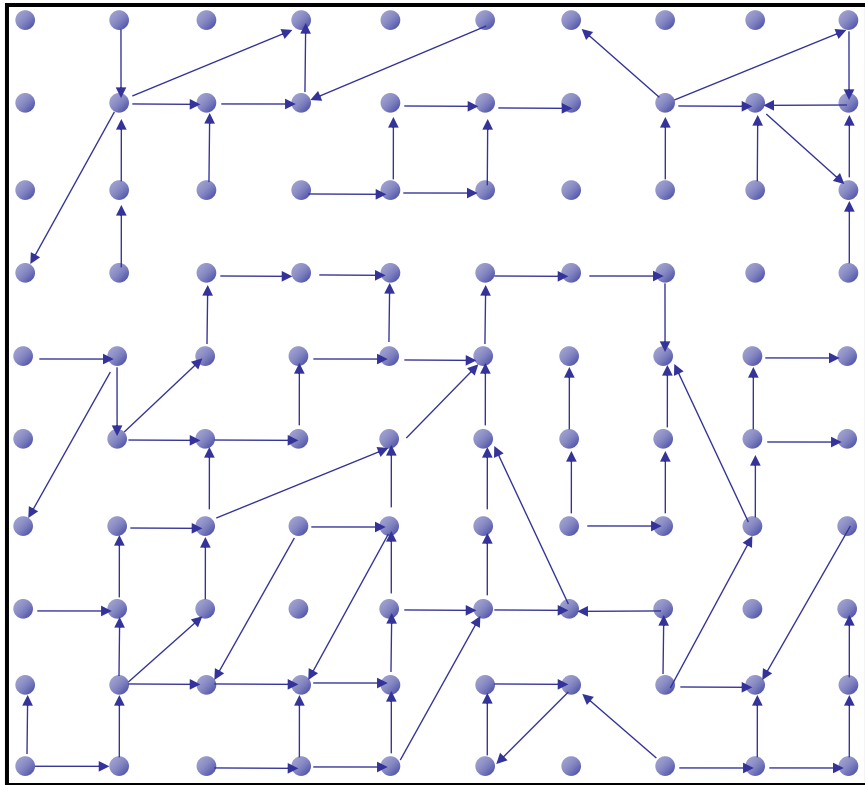
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        unlock ();  
        new ++;  
    }  
4: } while(new != old);  
5: unlock ();  
    return;  
}
```

```
Example ( ) {  
1: do{  
    if L=1 goto ERR;  
    else L=1;  
    old = new;  
    q = q->next;  
2:    if (q != NULL){  
3:        q->data = new;  
        if L=0 goto ERR;  
        else L=0;  
        new ++;  
    }  
4: } while(new != old);  
5: if L=0 goto ERR;  
    else L=0;  
    return;  
ERR: abort ();  
}
```



*Original program
violates spec iff
new program
reaches ERR*

Program As Labeled Transition System



State



Transition



$pc \mapsto 3$
 $lock \mapsto \bullet$
 $old \mapsto 5$
 $new \mapsto 5$
 $q \mapsto 0x133a$

```

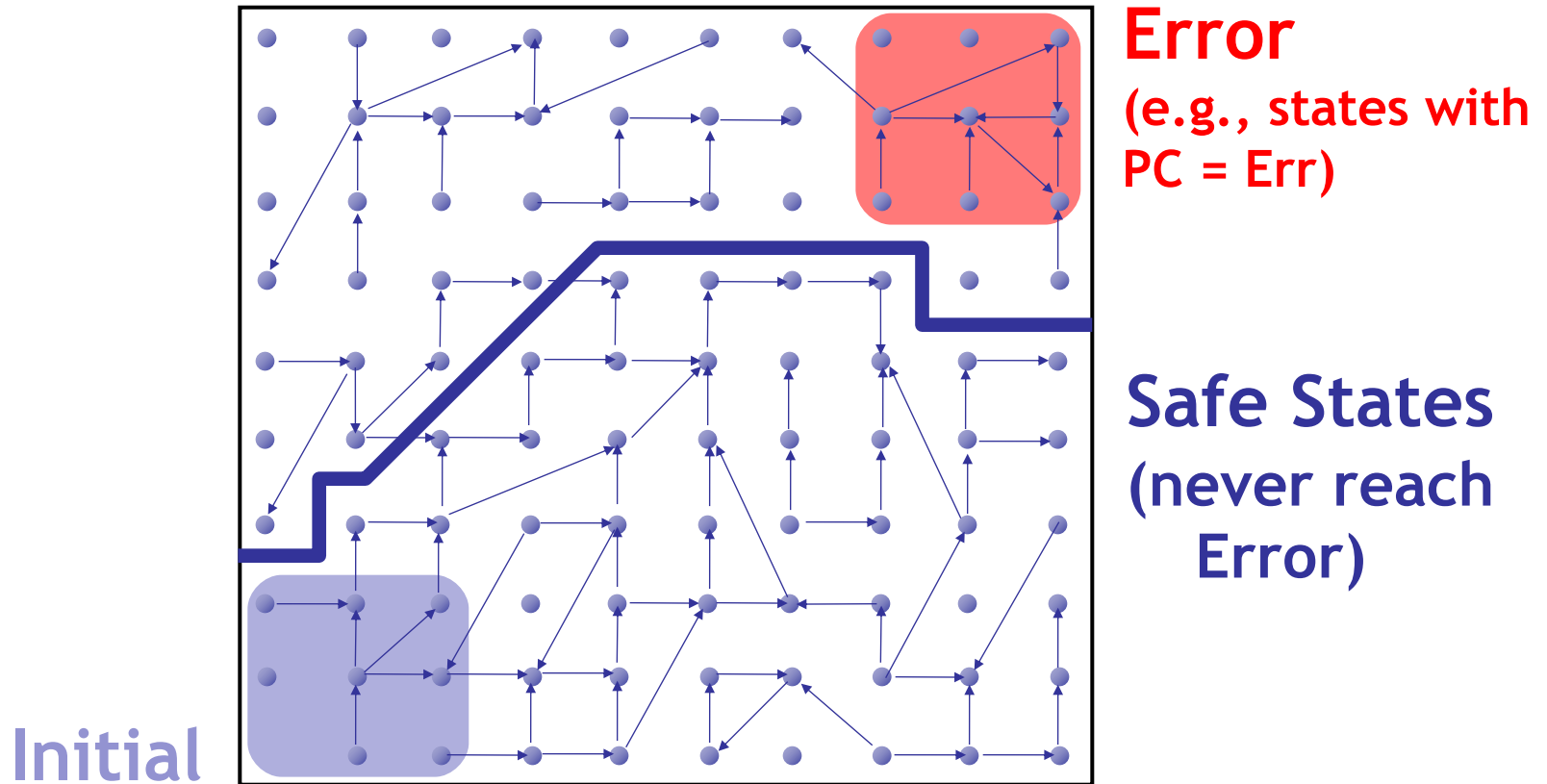
3: unlock ();
   new++;
4: } ...
    
```

$pc \mapsto 4$
 $lock \mapsto \circ$
 $old \mapsto 5$
 $new \mapsto 6$
 $q \mapsto 0x133a$

```

Example ( ) {
1: do {
    lock ();
    old = new;
    q = q->next;
2:   if (q != NULL){
3:     q->data = new;
       unlock ();
       new ++;
   }
4: } while(new != old);
5: unlock ();
   return; }
    
```


The Safety Verification Problem



Is there a **path** from an **initial** to an **error** state ?

Problem: Infinite state graph (old=1, old=2, old=...)

Solution : Set of states \simeq logical formula

Representing [Sets of States] as *Formulas*

$[F]$

states satisfying F $\{s \mid s \models F\}$

F

FO fmla over prog. vars

$[F_1] \cap [F_2]$

$F_1 \wedge F_2$

$[F_1] \cup [F_2]$

$F_1 \vee F_2$

$\overline{[F]}$

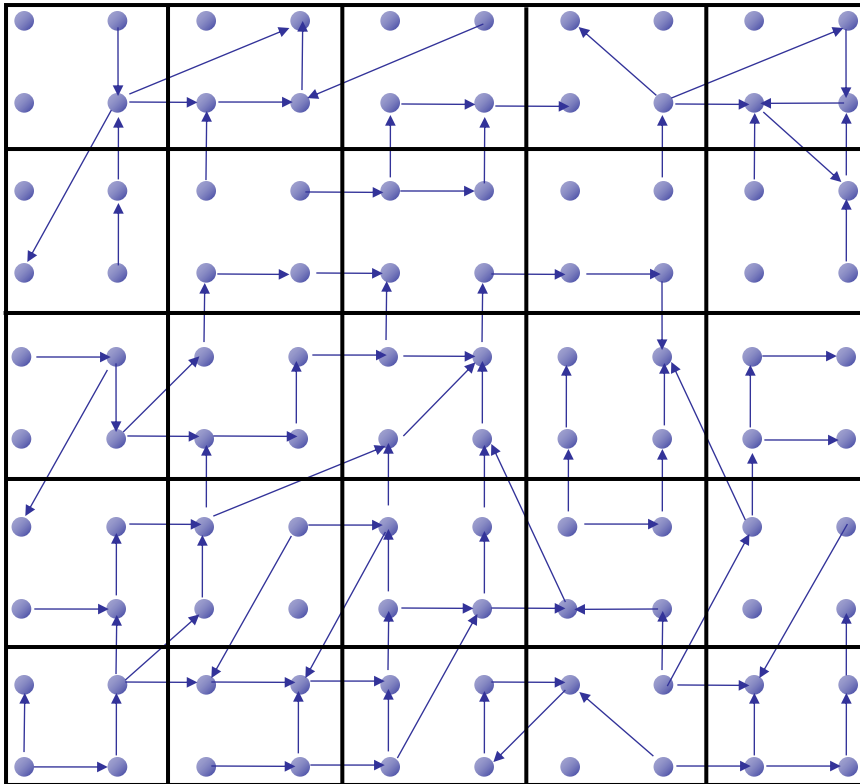
$\neg F$

$[F_1] \subseteq [F_2]$

$F_1 \Rightarrow F_2$

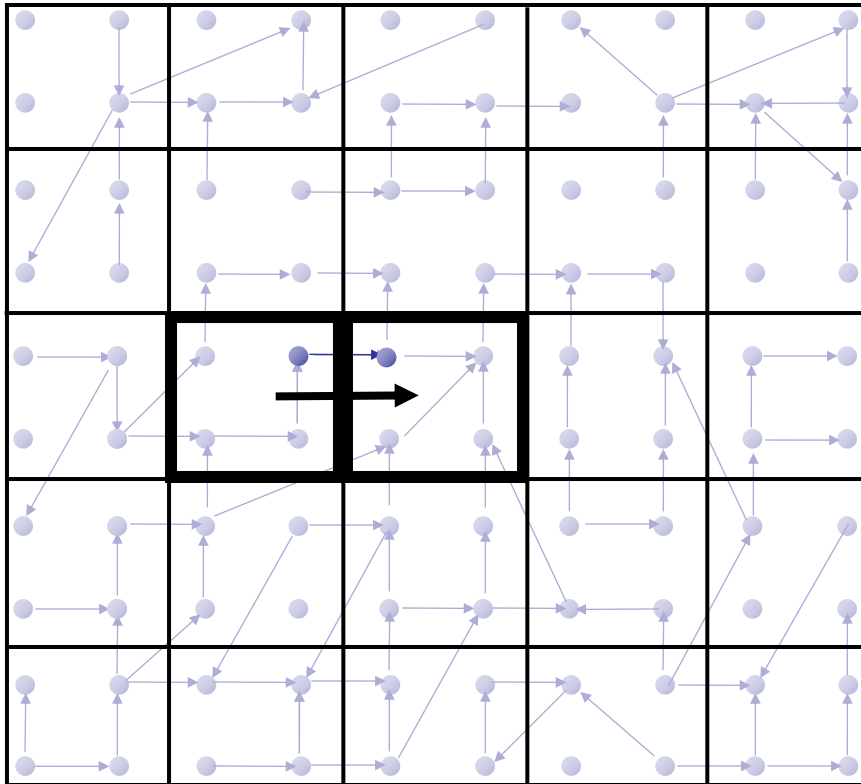
i.e. $F_1 \wedge \neg F_2$ unsatisfiable

Idea 1: Predicate Abstraction

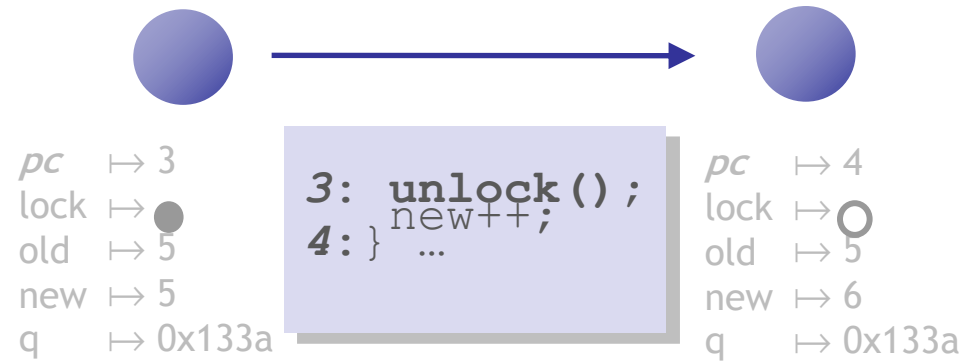


- **Predicates** on program state:
lock (i.e., *lock=true*)
old = new
- States satisfying **same** predicates are **equivalent**
 - Merged into one **abstract state**
- #abstract states is **finite**
 - **Thus model-checking the abstraction will be feasible!**

Abstract States and Transitions



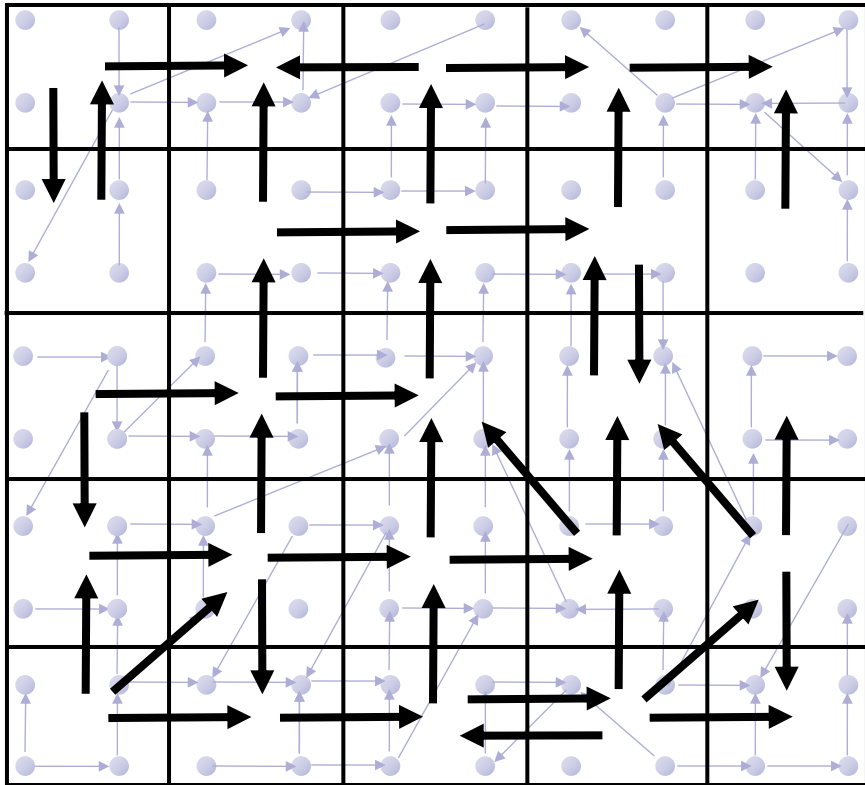
State



lock
old=new

$\neg lock$
 $\neg old=new$

Abstraction



Existential Lifting
 (i.e., $A_1 \rightarrow A_2$ iff $\exists c_1 \in A_1. \exists c_2 \in A_2. c_1 \rightarrow c_2$)

State

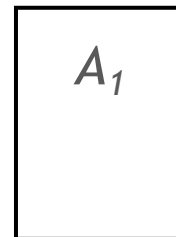


$pc \mapsto 3$
 $lock \mapsto \bullet$
 $old \mapsto 5$
 $new \mapsto 5$
 $q \mapsto 0x133a$

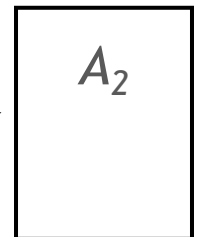
```

3: unlock ();
   new++;
4: } ...
    
```

$pc \mapsto 4$
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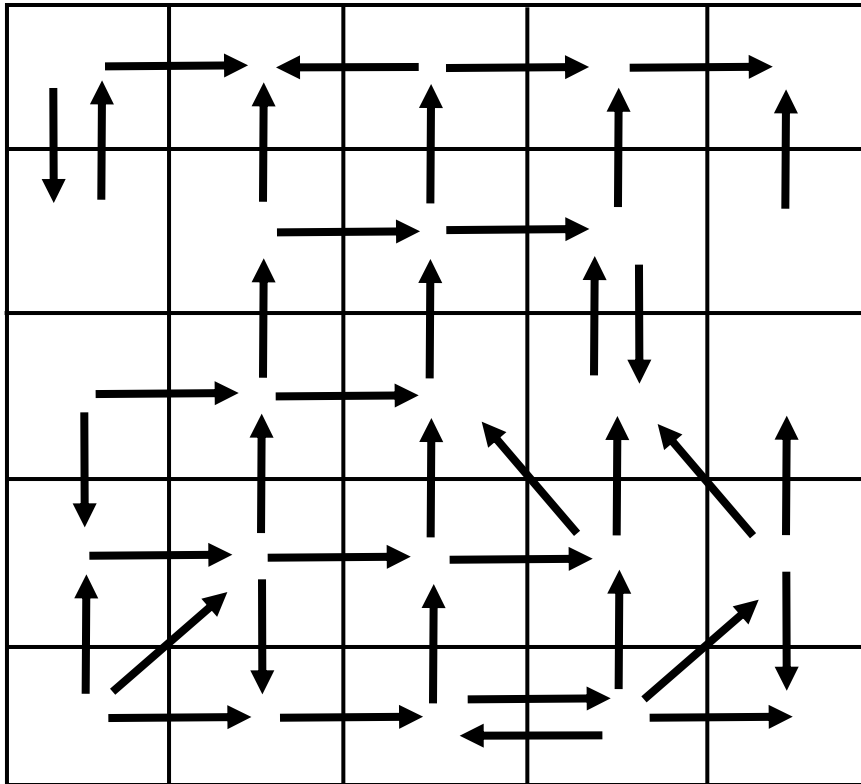
Theorem Prover



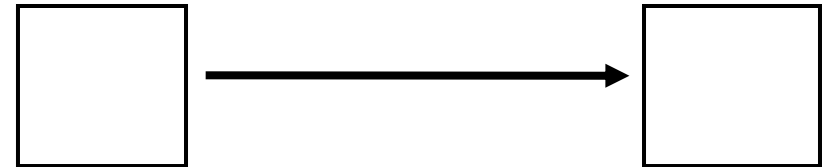
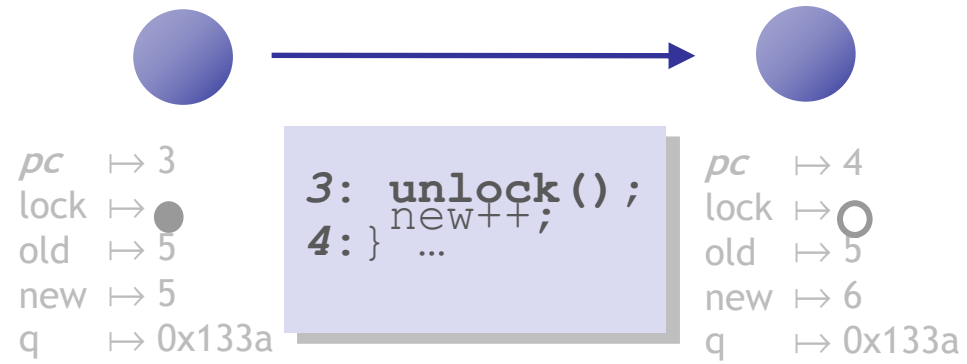
$lock$
 $old=new$

$\neg lock$
 $\neg old=new$

Abstraction



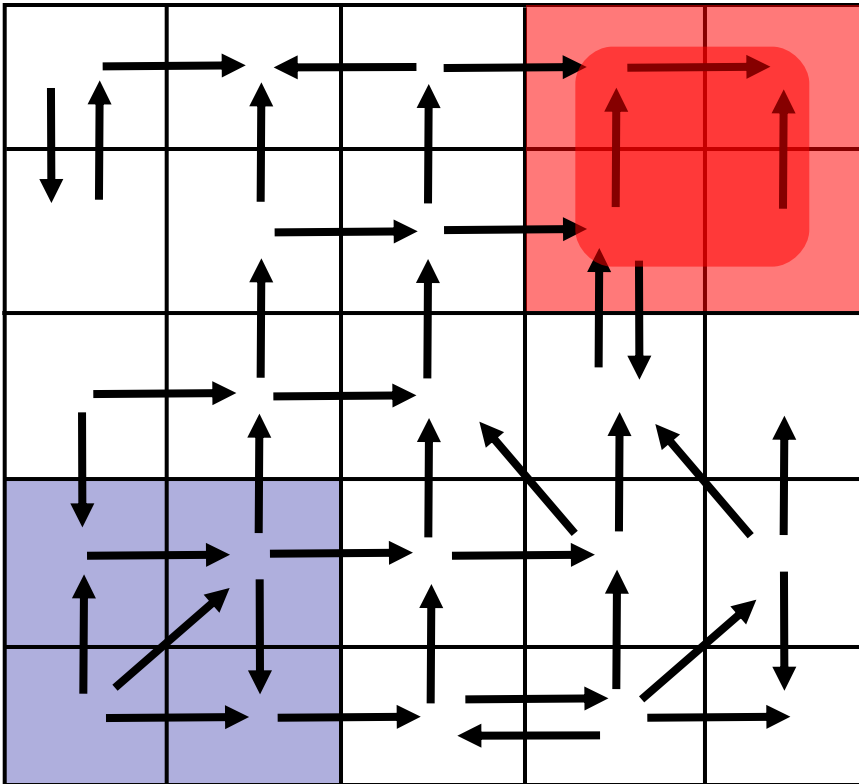
State



lock
old=new

\rightarrow *lock*
 \rightarrow *old=new*

Analyze Abstraction



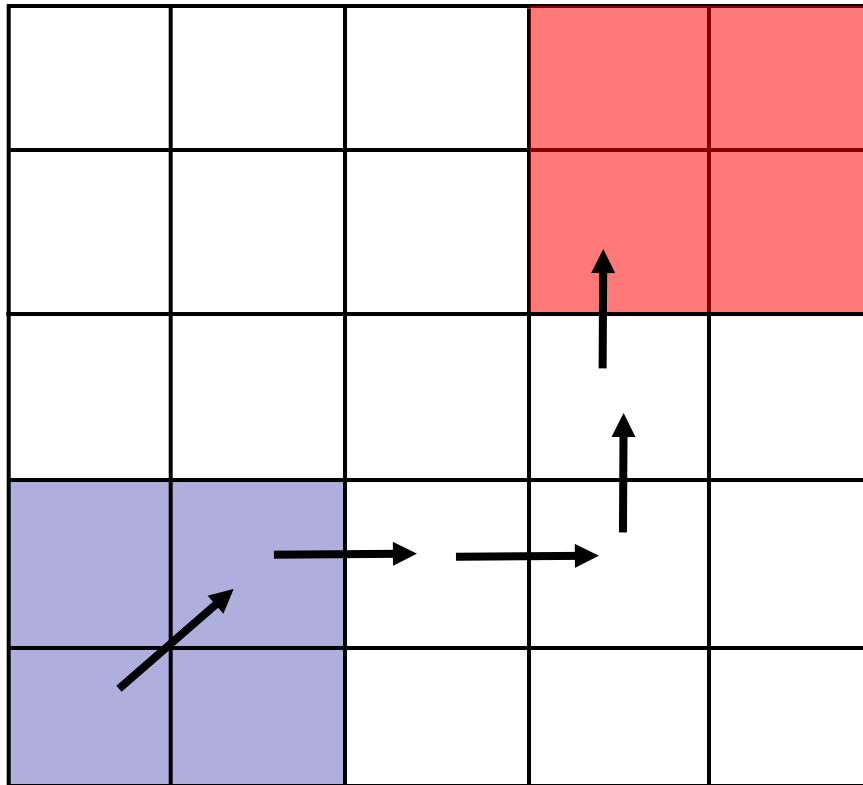
Analyze finite graph

Over Approximate:
Safe \Rightarrow System Safe
No **false negatives**

Problem

Spurious **counterexamples**

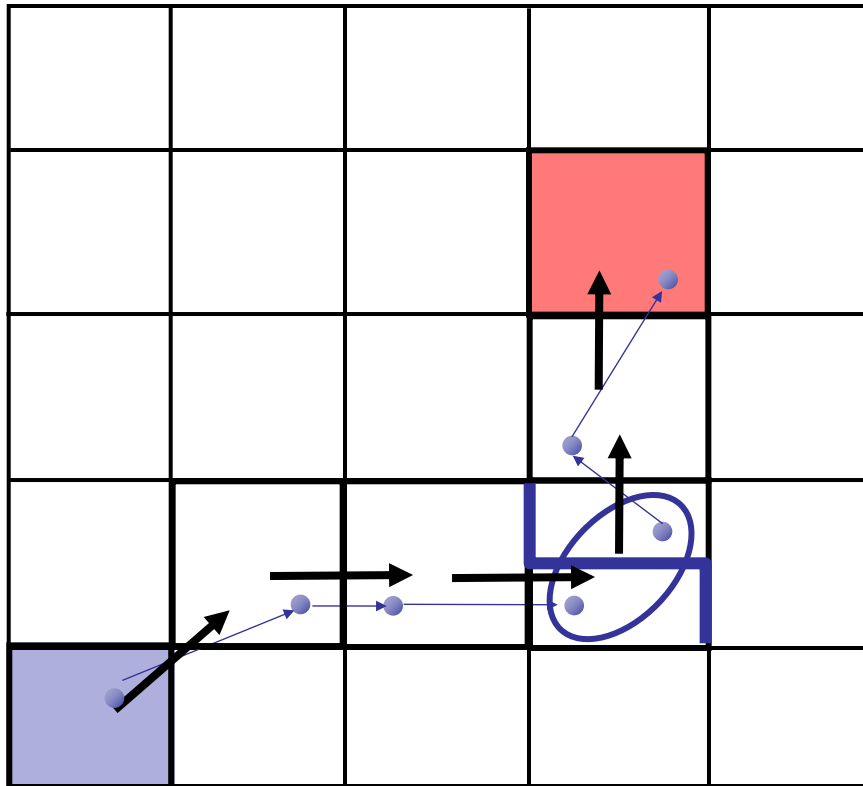
Idea 2: Counterex.-Guided Refinement



Solution

Use spurious **counterexamples** to **refine** abstraction!

Idea 2: Counterex.-Guided Refinement

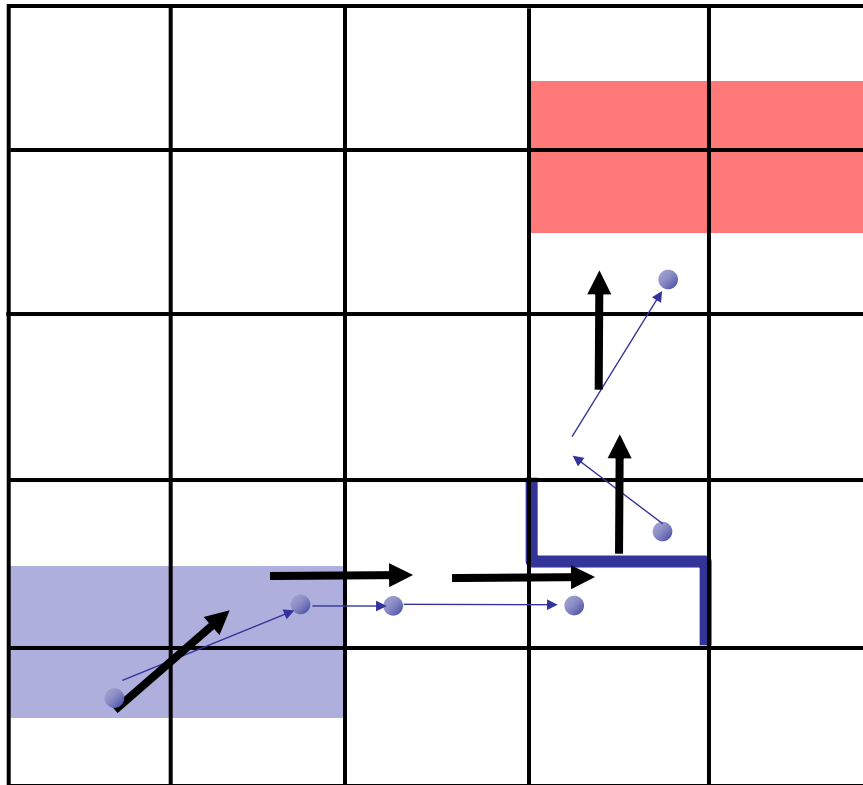


Solution

Use spurious **counterexamples** to **refine** abstraction

1. **Add predicates** to distinguish states across **cut**
2. Build **refined** abstraction
Imprecision due to **merge**

Iterative Abstraction-Refinement



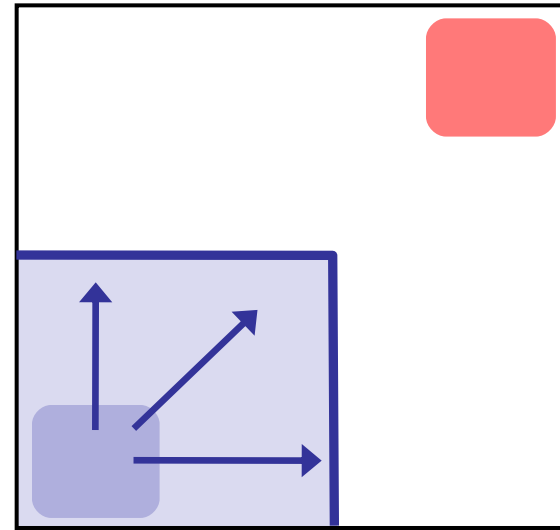
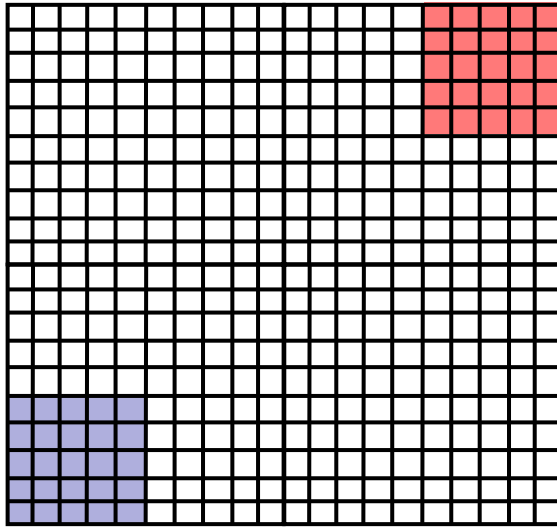
Solution

Use spurious **counterexamples** to **refine** abstraction

1. Add predicates to distinguish states across **cut**
2. Build **refined** abstraction
-eliminates counterexample
3. **Repeat** search
Untill real counterexample or system proved safe

[Kurshan et al 93] [Clarke et al 00]
[Ball-Rajamani 01]

Problem: Abstraction is Expensive



Reachable

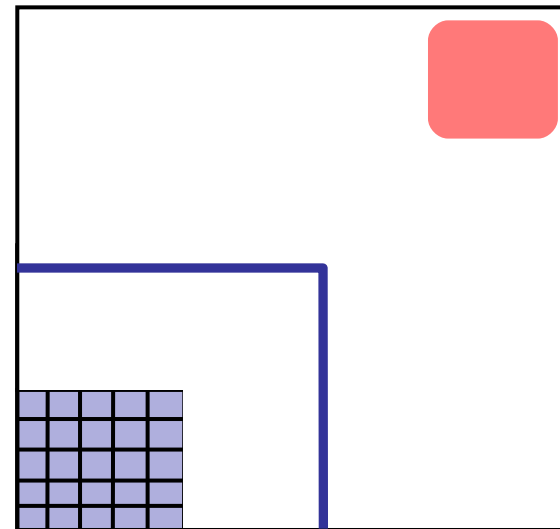
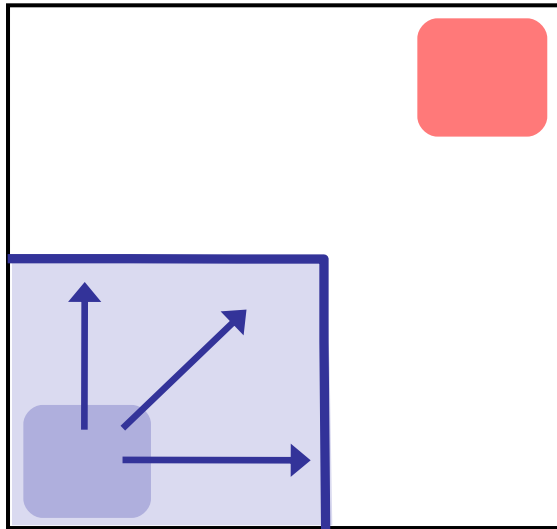
Problem

#abstract states = $2^{\text{\#predicates}}$
Exponential Thm. Prover queries

Observe

Fraction of state space reachable
#Preds ~ 100's, #States ~ 2^{100} ,
#Reach ~ 1000's

Solution 1: Only Abstract Reachable States



Safe

Problem

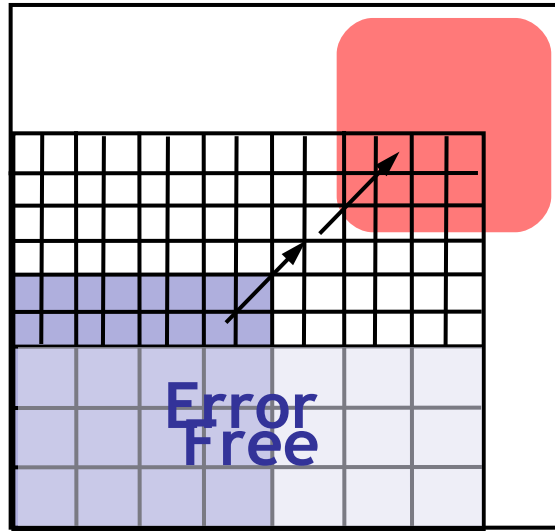
#abstract states = $2^{\text{\#predicates}}$

Exponential Thm. Prover queries

Solution

Build abstraction **during** search

Solution2: Don't Refine Error-Free Regions



Problem

#abstract states = $2^{\text{\#predicates}}$
Exponential Thm. Prover queries

Solution

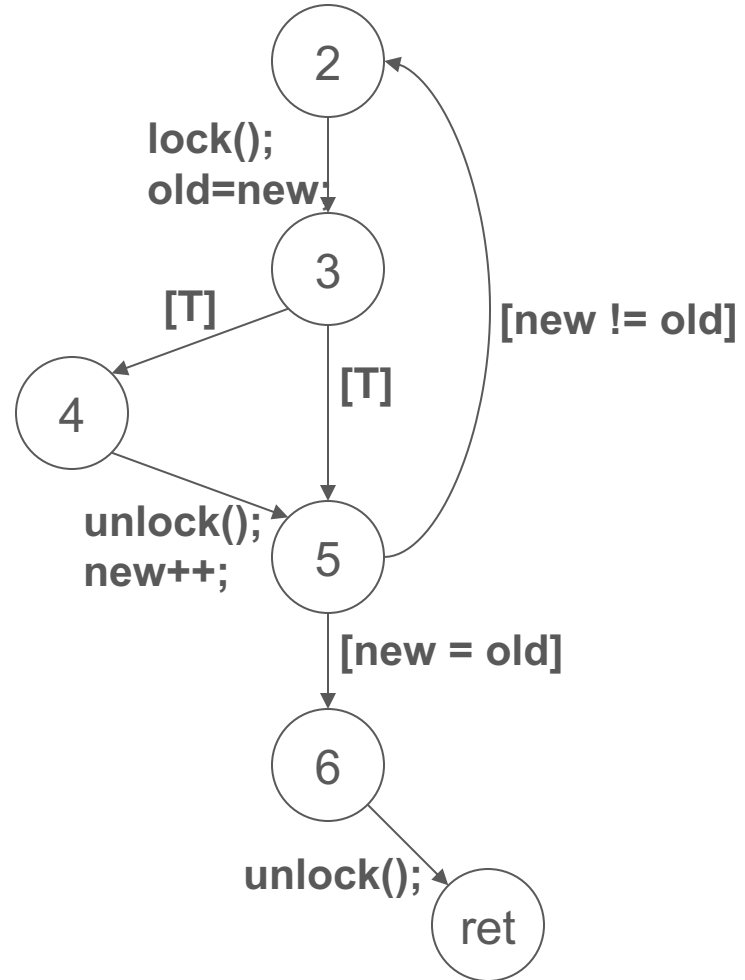
Don't refine error-free regions

Build reachability tree.

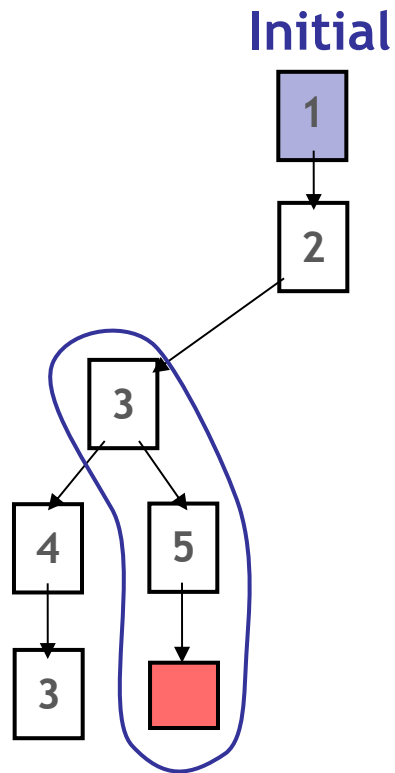
- Generate Abstract Reachability Tree
 - Contains all reachable nodes
 - Annotates each node with state
 - Initially LOCK = 0 or LOCK = 1
 - Cross product of CFA and data flow abstraction
- Algorithm: depth-first search
 - Generate nodes one by one
 - If you come to a node that's already in the tree, stop
 - This state has already been explored through a different control flow path
 - If you come to an error node, stop

Less abstractly: build reachability tree

```
2:  do {
      lock();
      old = new;
3:    if (*){
4:      unlock();
      new++;
    }
5:  } while (new != old);
6:  unlock();
   return;
```



Key Idea: Reachability Tree



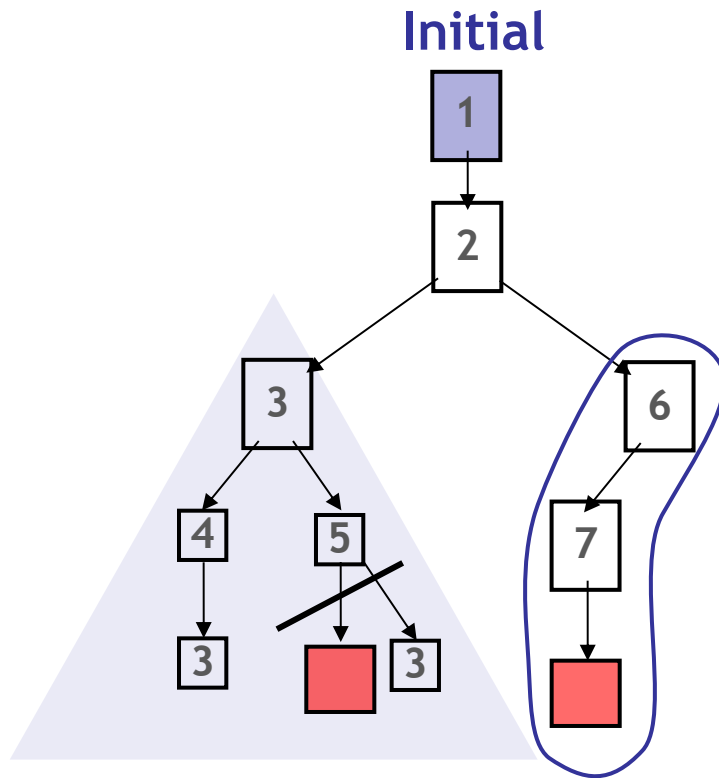
Unroll Abstraction

1. Pick tree-node (=abs. state)
2. Add children (=abs. successors)
3. On **re-visiting** abs. state, **cut-off**

Find min infeasible suffix

- Learn new predicates
- Rebuild subtree with new preds.

Key Idea: Reachability Tree



Error Free

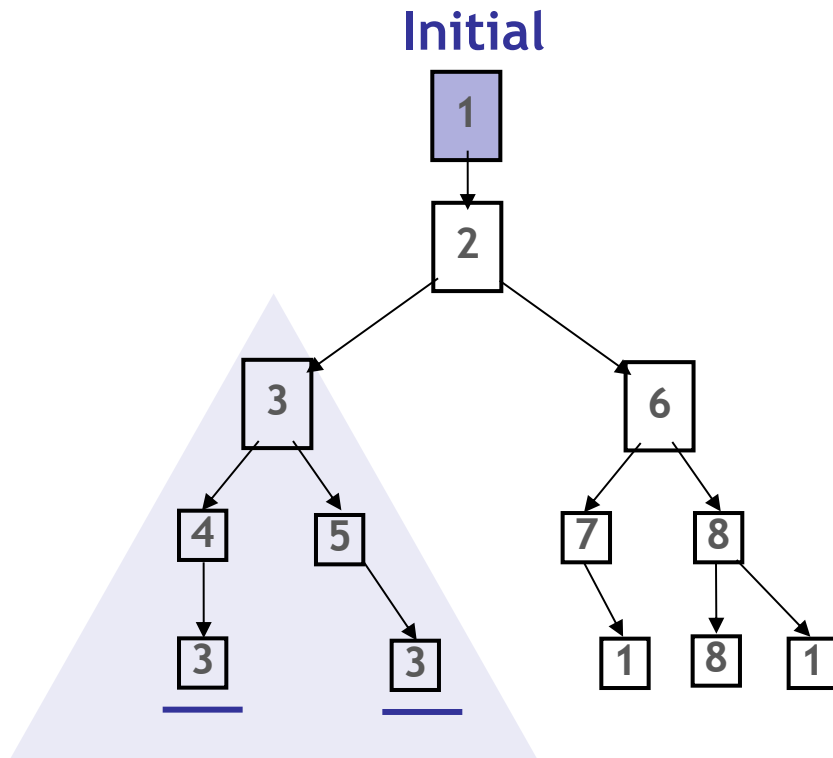
Unroll Abstraction

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Key Idea: Reachability Tree



Unroll

1. Pick tree-node (=abs. state)
2. Add children (=abs. successors)
3. On **re-visiting** abs. state, **cut-off**

Find min spurious suffix

- Learn new predicates
- Rebuild subtree with new preds.

Error Free

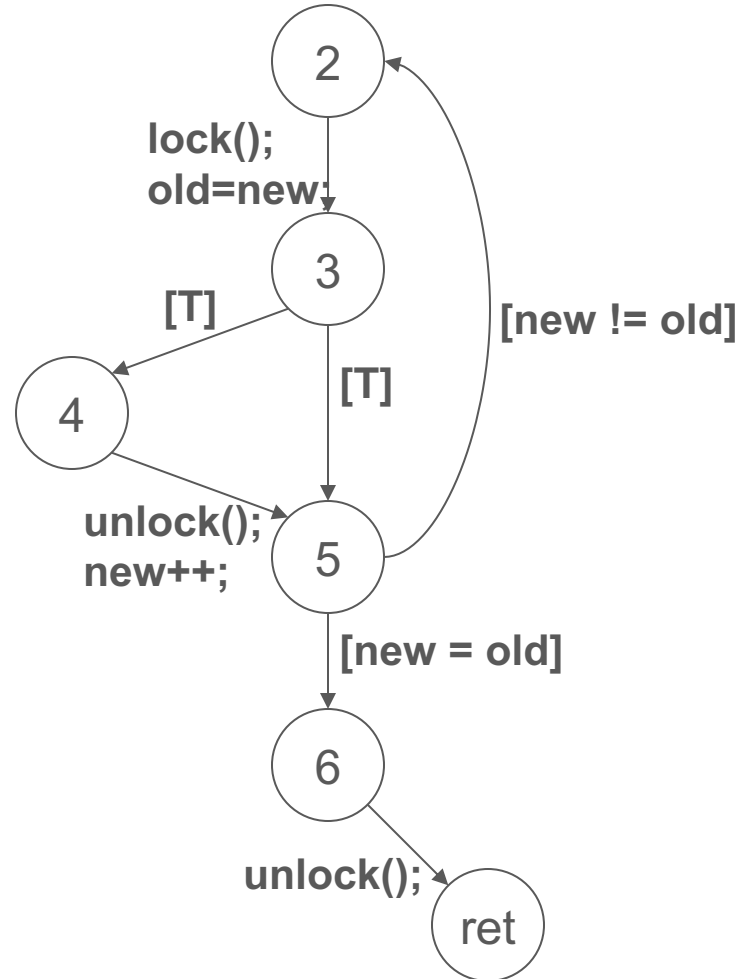
SAFE

S1: Only Abstract Reachable States

S2: Don't refine error-free regions

Less abstractly: build reachability tree

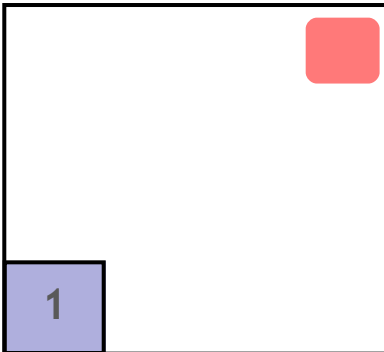
```
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    lock();  
    old = new;  
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      new++;  
    }  
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   return;
```



Build-and-Search

```
Example ( ) {  
1: do{  
    lock ();  
    old = new;  
    q = q->next;  
2:   if (q != NULL){  
3:     q->data = new;  
     unlock ();  
     new ++;  
   }  
4: }while(new != old);  
5: unlock ();  
}
```

1 → LOCK



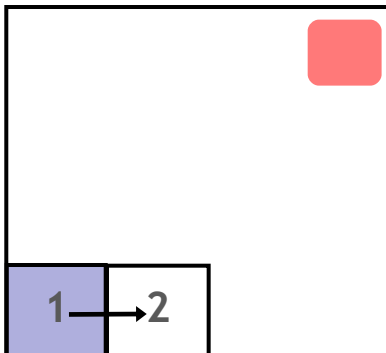
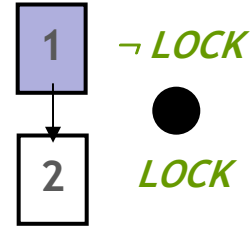
Predicates: LOCK

Reachability Tree

Build-and-Search

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    }  
4: }while(new != old);  
5: unlock ();  
}
```

lock ()
old = new
q=q->next

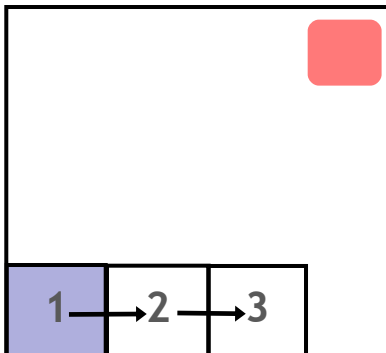
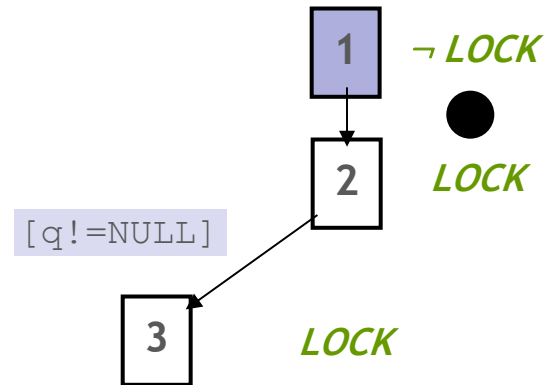


Predicates: LOCK

Reachability Tree

Build-and-Search

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    }  
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}
```



Predicates: *LOCK*

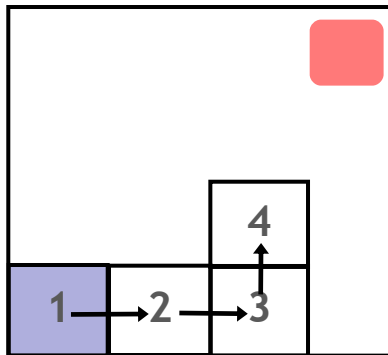
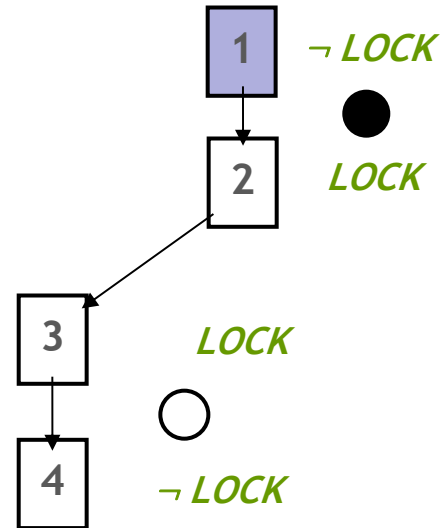
Reachability Tree

Build-and-Search

```

Example ( ) {
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3:     q->data = new;
    unlock ();
    new ++;
    }
4: }while(new != old);
5: unlock ();
}
    
```

`q->data = new`
`unlock ()`
`new++`

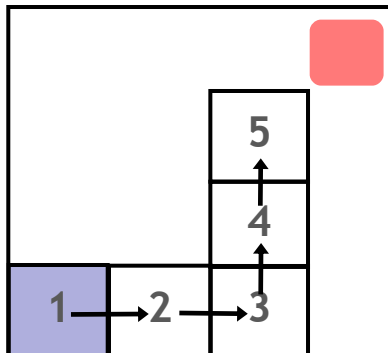


Predicates: $LOCK$

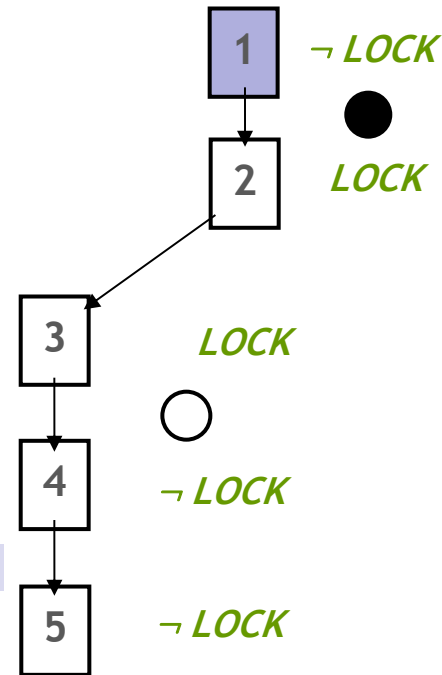
Reachability Tree

Build-and-Search

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4: }while(new != old);  
5: unlock ();  
}
```



Predicates: *LOCK*

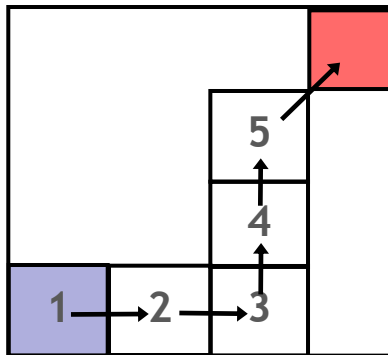


Reachability Tree

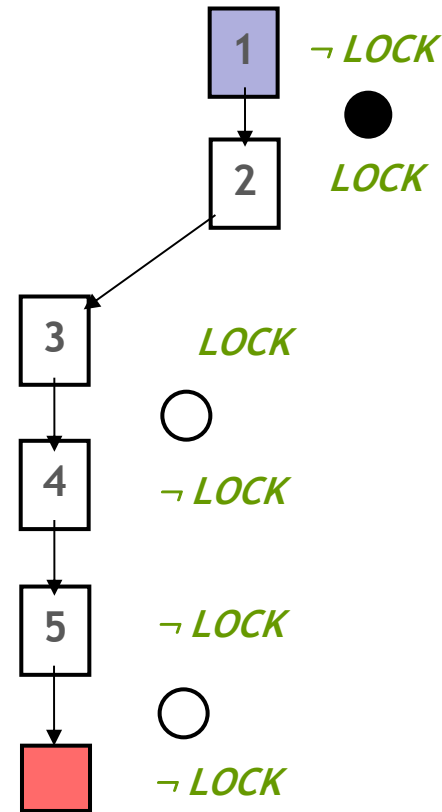
Build-and-Search

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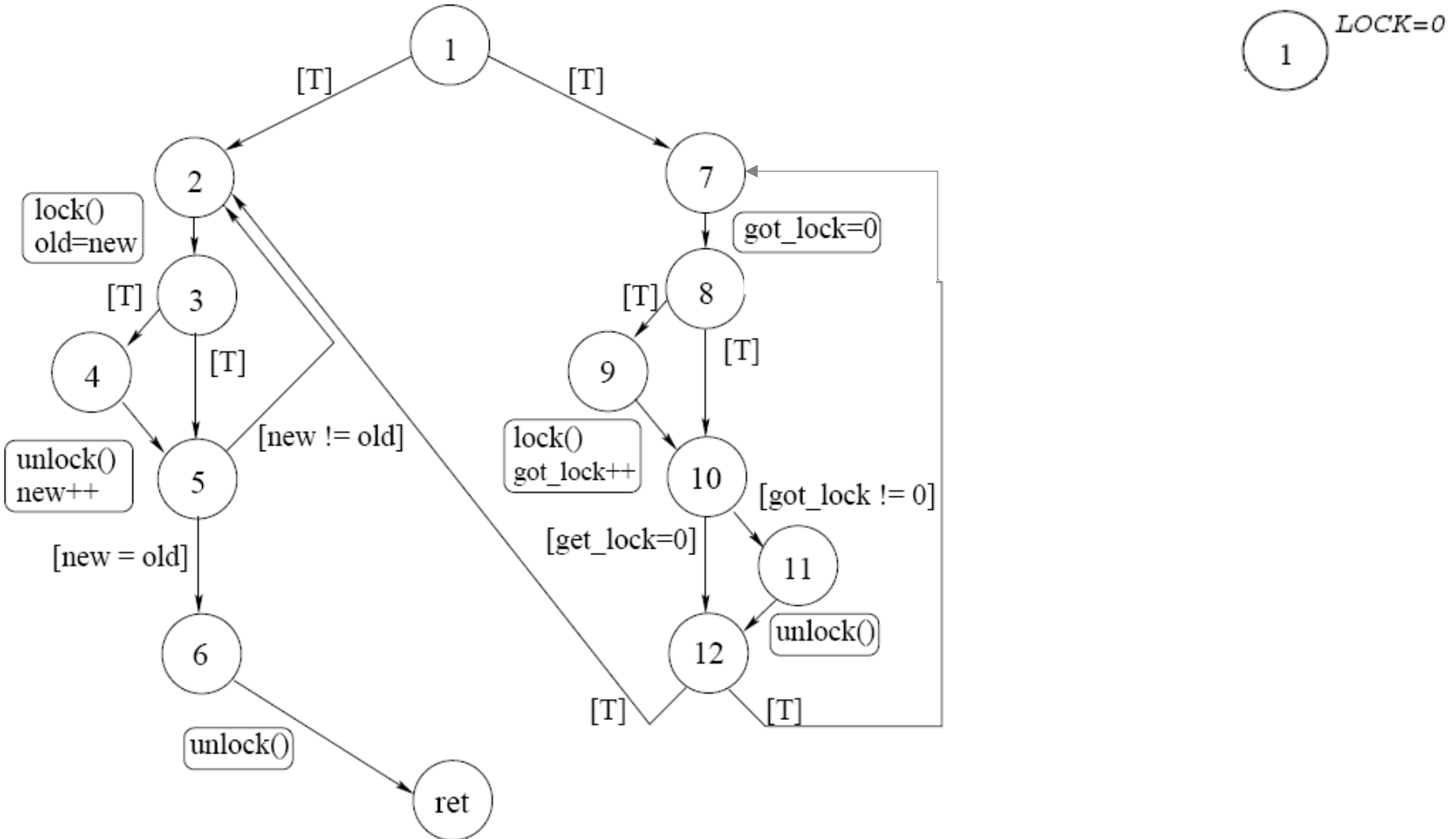


Predicates: *LOCK*



Reachability Tree

Depth First Search Example

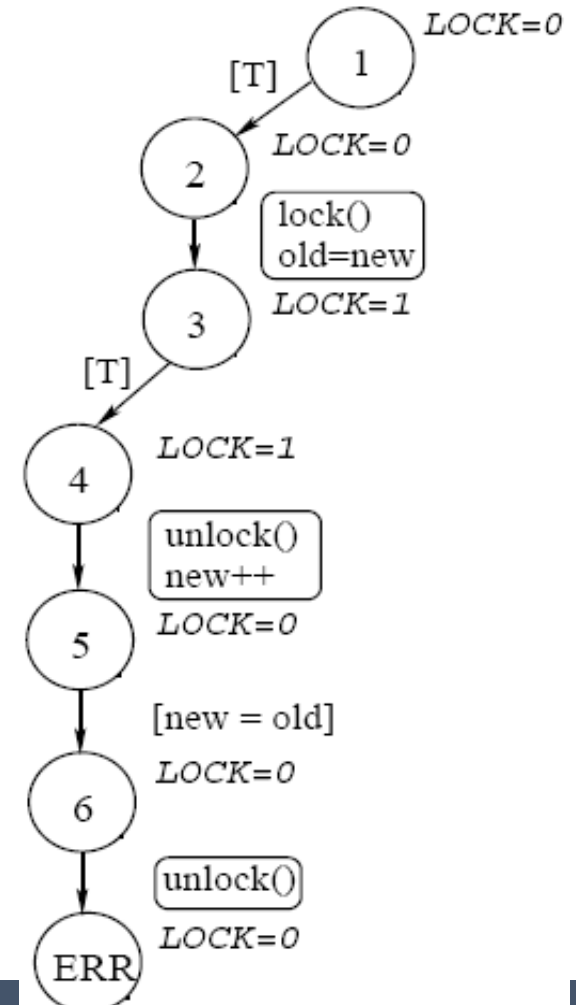


Is the Error Real?

- Use weakest preconditions to find out the weakest precondition that leads to the error
 - If the weakest precondition is false, there is no initial program condition that can lead to the error
 - Therefore the error is spurious
- Blast uses a variant of weakest preconditions
 - creates a new variable for each assignment before using weakest preconditions
 - Instead of substituting on assignment, adds new constraint
 - Helps isolate the reason for the spurious error more effectively

Is the Error Real?

- assume True;
- lock();
- old = new;
- assume True;
- unlock();
- new++;
- assume new==old
- error (lock==0)



Model Locking as Assignment

- `assume True;`
- `lock = 1;`
- `old = new;`
- `assume True;`
- `lock = 0;`
- `new = new + 1;`
- `assume new==old`
- `error (lock==0)`

Index the Variables

- `assume True;`
- `lock1 = 1`
- `old1 = new1;`
- `assume True;`
- `lock2 = 0`
- `new2 = new1 + 1`
- `assume new2==old1`
- `error (lock2==0)`

Generate Weakest Preconditions

- assume True;
- lock1 = 1
- old1 = new1;
- assume True;
- lock2 = 0
- new2 = new1 + 1
- assume new2==old1
- error (lock2==0)

\wedge True
 \wedge lock1==1
 \wedge old1==new1
 \wedge True
 \wedge lock2==0
 \wedge new2==new1+1
 \wedge new2==old1
lock2==0

Contradictory!



Relevant Sidebar: Craig Interpolation

- Given an unsatisfiable formula $A \wedge B$, the Craig Interpolant I is a formula such that:
 - $A \rightarrow I$
 - $I \wedge B$ is unsatisfiable
 - I only refers to variables mentioned in both A and B
- It is guaranteed to exist, proof elided.
- $\wedge \text{True}$
- $\wedge \text{lock1}==1$
- $\wedge \text{old1}==\text{new1}$
- $\wedge \text{True}$
- $\wedge \text{lock2}==0$
- $\wedge \text{new2}==\text{new1}+1$
- $\wedge \text{new2}==\text{old1}$
- $\text{lock2}==0$

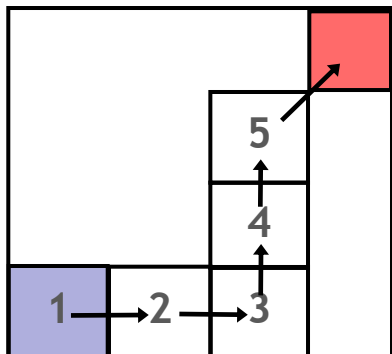
Why is the Error Spurious?

- More precisely, what predicate could we track that would eliminate the spurious error message?
 - Consider, for each node, the constraints generated before that node ($c1$) and after that node ($c2$)
 - Find a condition I such that
 - $c1 \Rightarrow I$
 - I is true at the node
 - I only contains variables mentioned in both $c1$ and $c2$
 - I mentions only variables in scope (not old or future copies)
 - $I \wedge c2 = \text{false}$
 - I is enough to show that the rest of the path is infeasible
 - I is guaranteed to exist
 - See Craig Interpolation
- $\wedge \text{True}$
 - $\wedge \text{lock1} == 1$
 - $\wedge \text{old1} == \text{new1}$ ← Interpolant: $\text{old} == \text{new}$
 - $\wedge \text{True}$
 - $\wedge \text{lock2} == 0$
 - $\wedge \text{new2} == \text{new1} + 1$
 - $\wedge \text{new2} == \text{old1}$
 - $\text{lock2} == 0$

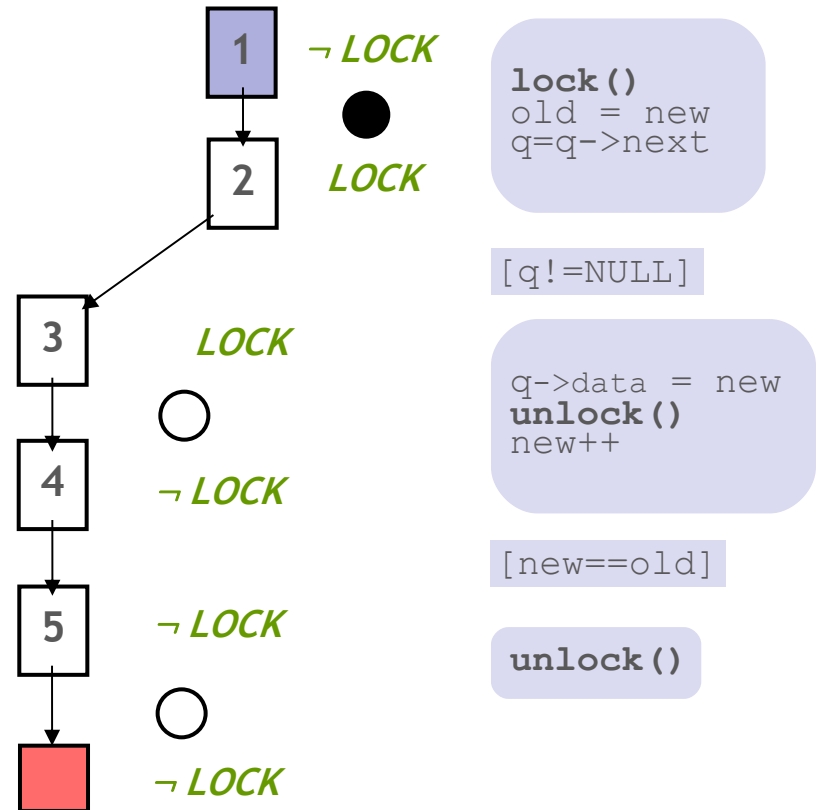
Analyze Counterexample

```

Example ( ) {
1: do{
    lock ();
    old = new;
    q = q->next;
2:   if (q != NULL){
3:     q->data = new;
    unlock ();
    new ++;
    }
4: }while(new != old);
5: unlock ();
}
    
```



Predicates: *LOCK*

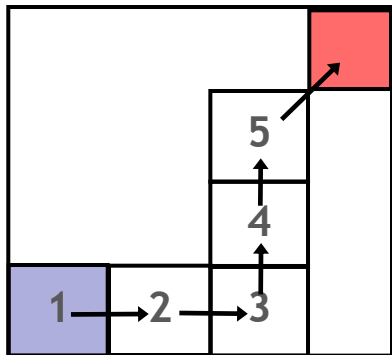


Reachability Tree

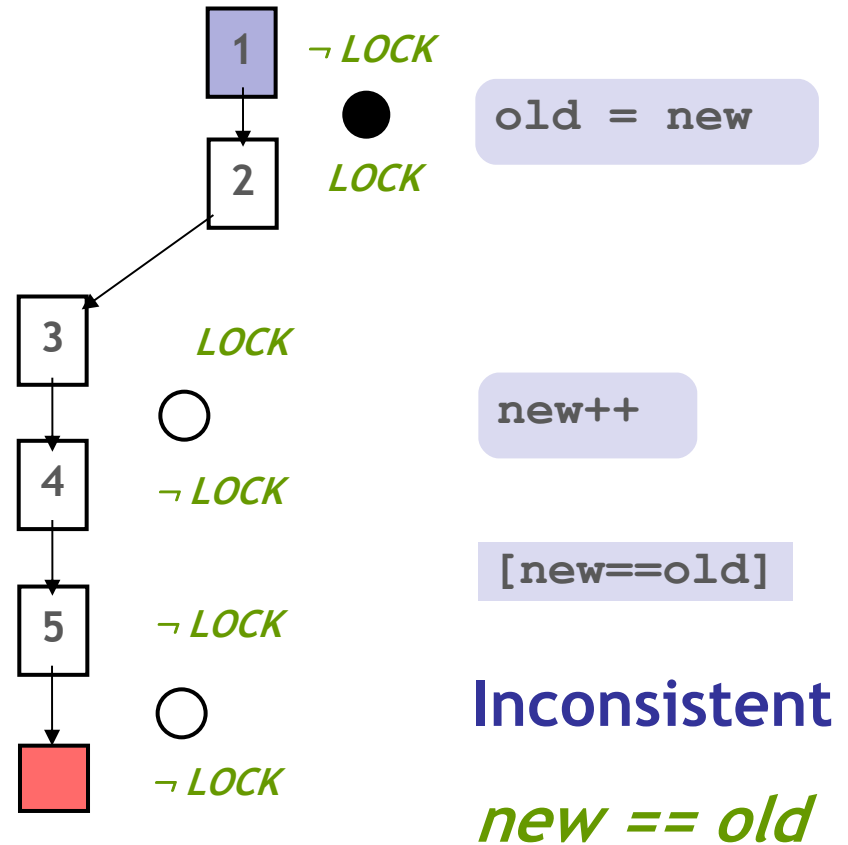
Analyze Counterexample

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Example ( ) {
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2:   if (q != NULL){
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    new ++;
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}
    
```



Predicates: *LOCK*

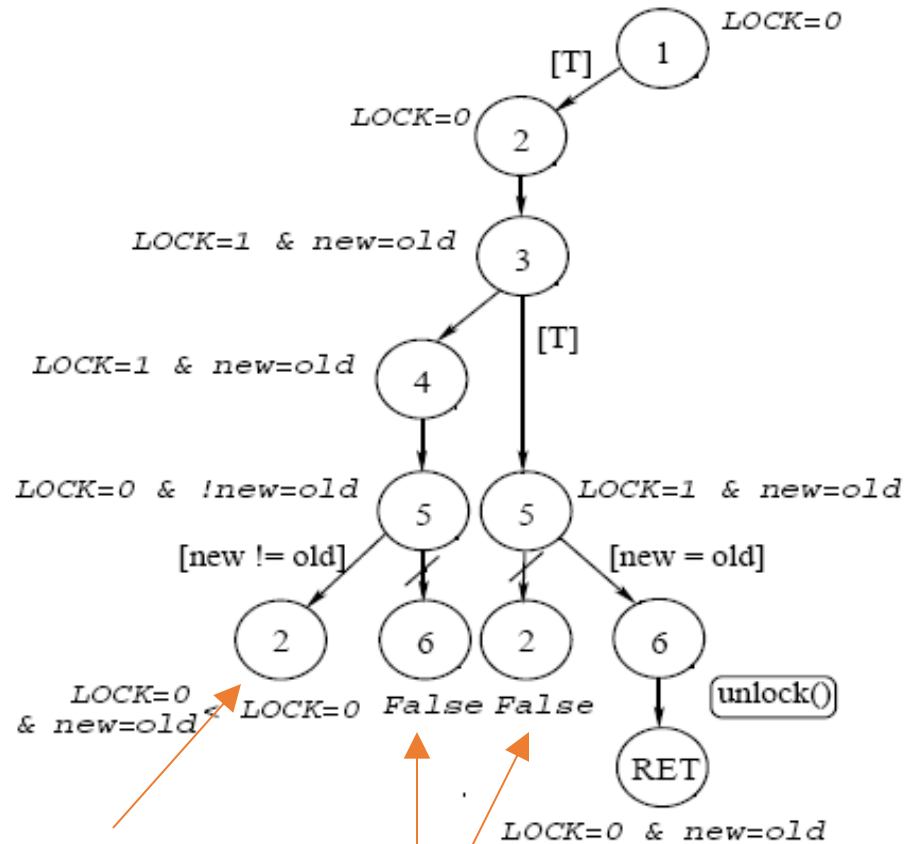
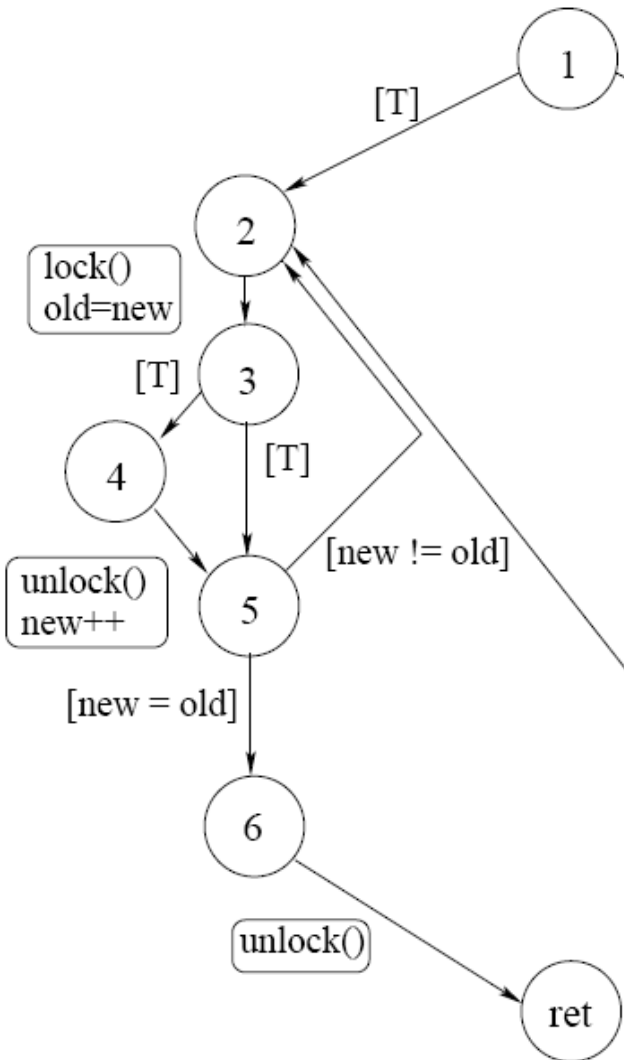


Reachability Tree

Reanalyzing the Program

- Explore a subtree again
 - Start where new predicates were discovered
 - This time, track the new predicates
 - If the conjunction of the predicates on a node is false, stop exploring—this node is unreachable

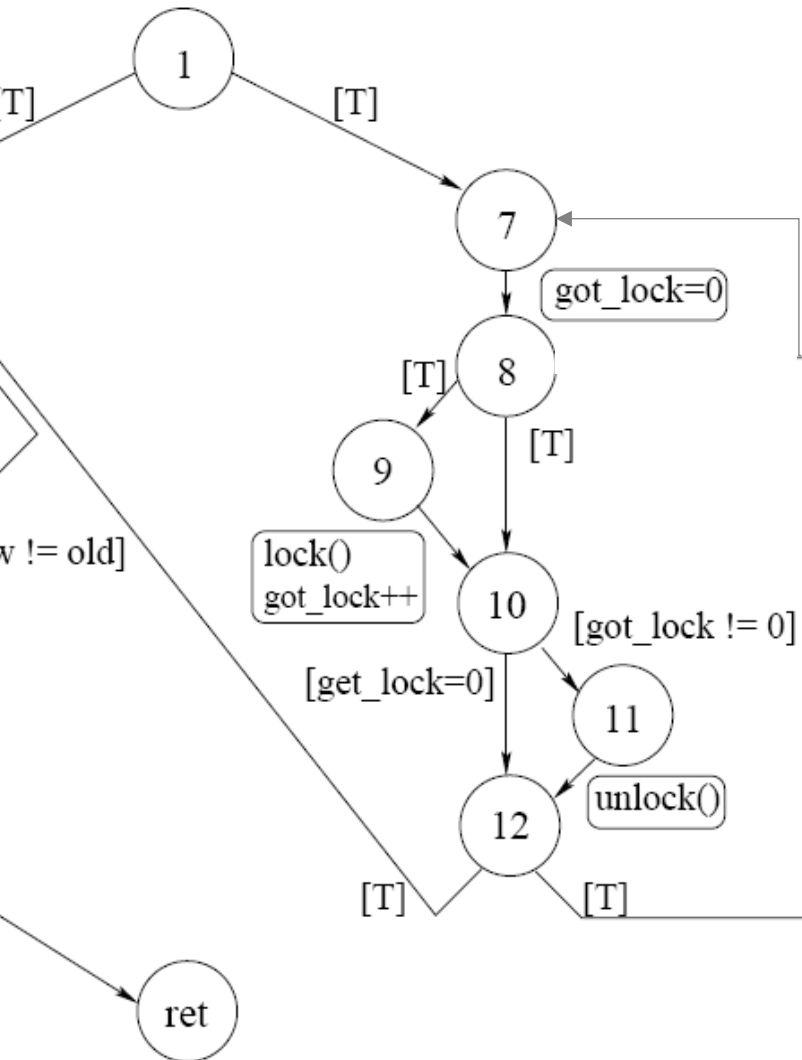
Reanalysis Example



Already Covered

Unreachable

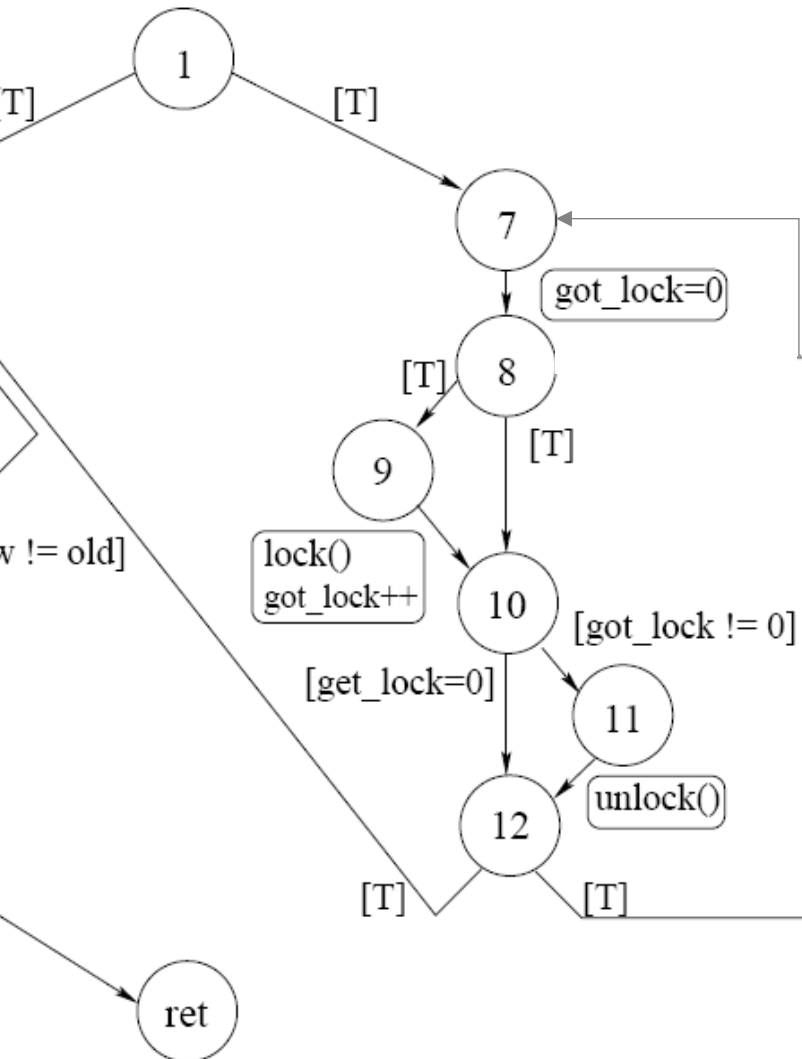
Analyzing the Right Hand Side



Exercise: run weakest preconditions from the `unlock()` at the end of the path 1-7-8-10-11-12.

Recall that we model locking with a variable *lock*, so `unlock()` is an error if `lock==0`

Reanalysis



```

Example() {
1:  if (*) {
7:    do {
        got_lock = 0;
8:    if (*) {
9:      lock();
        got_lock++;
    }
10:   if (got_lock) {
11:     unlock();
    }
12:  } while (*)
}
  
```

Generate Weakest Preconditions

- `assume True;`
- `got_lock = 0;`
- `assume True;`
- `assume got_lock != 0;`
- `error (lock==0)`

Why is the Error Spurious?

- More precisely, what predicate could we track that would eliminate the spurious error message?
- Consider, for each node, the constraints generated before that node ($c1$) and after that node ($c2$)
- Find a condition I such that
 - $c1 \Rightarrow I$
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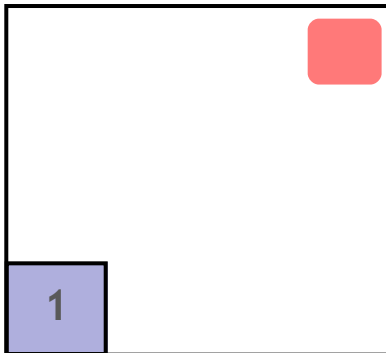
- $\wedge \text{True}$
- $\wedge \text{got_lock} == 0$
- $\wedge \text{True}$
- $\wedge \text{got_lock} != 0$
- $\text{lock} == 0$

Exercise: now find the Craig interpolant

Repeat Build-and-Search

```
Example ( ) {  
1: do{  
    lock ();  
    old = new;  
    q = q->next;  
2:   if (q != NULL){  
3:     q->data = new;  
     unlock ();  
     new ++;  
    }  
4: }while(new != old);  
5: unlock ();  
}
```

1 → LOCK



...but only at the minimum suffix!

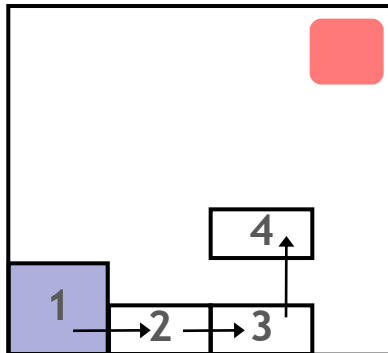
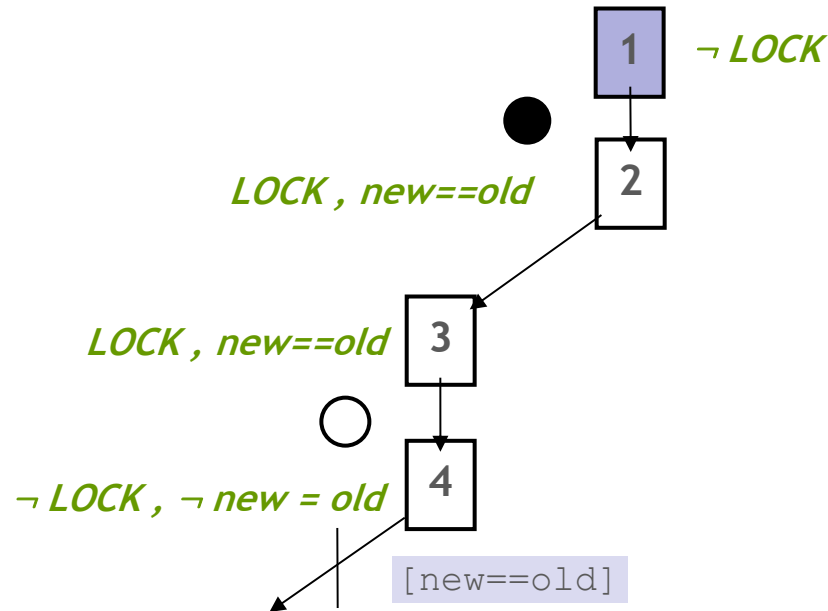
Reachability Tree

Predicates: *LOCK, new==old*

Repeat Build-and-Search

```

Example ( ) {
1: do{
    lock ();
    old = new;
    q = q->next;
2:   if (q != NULL){
3:     q->data = new;
    unlock ();
    new ++;
    }
4: }while(new != old);
5: unlock ();
}
    
```



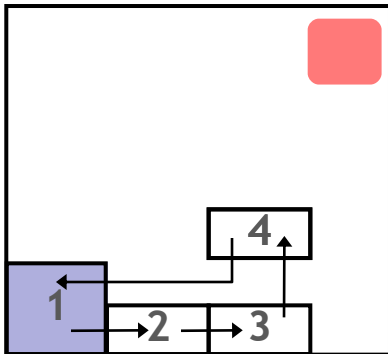
Predicates: $LOCK, new == old$

Reachability Tree

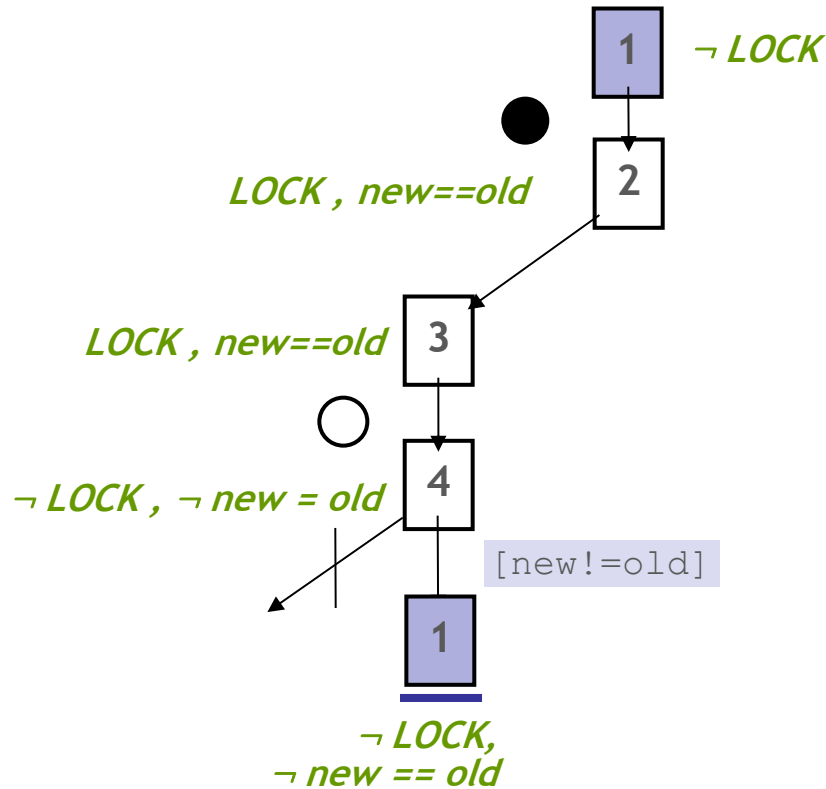
Repeat Build-and-Search

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    }
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}
    
```



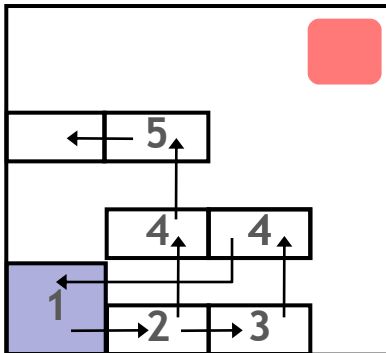
Predicates: *LOCK, new==old*



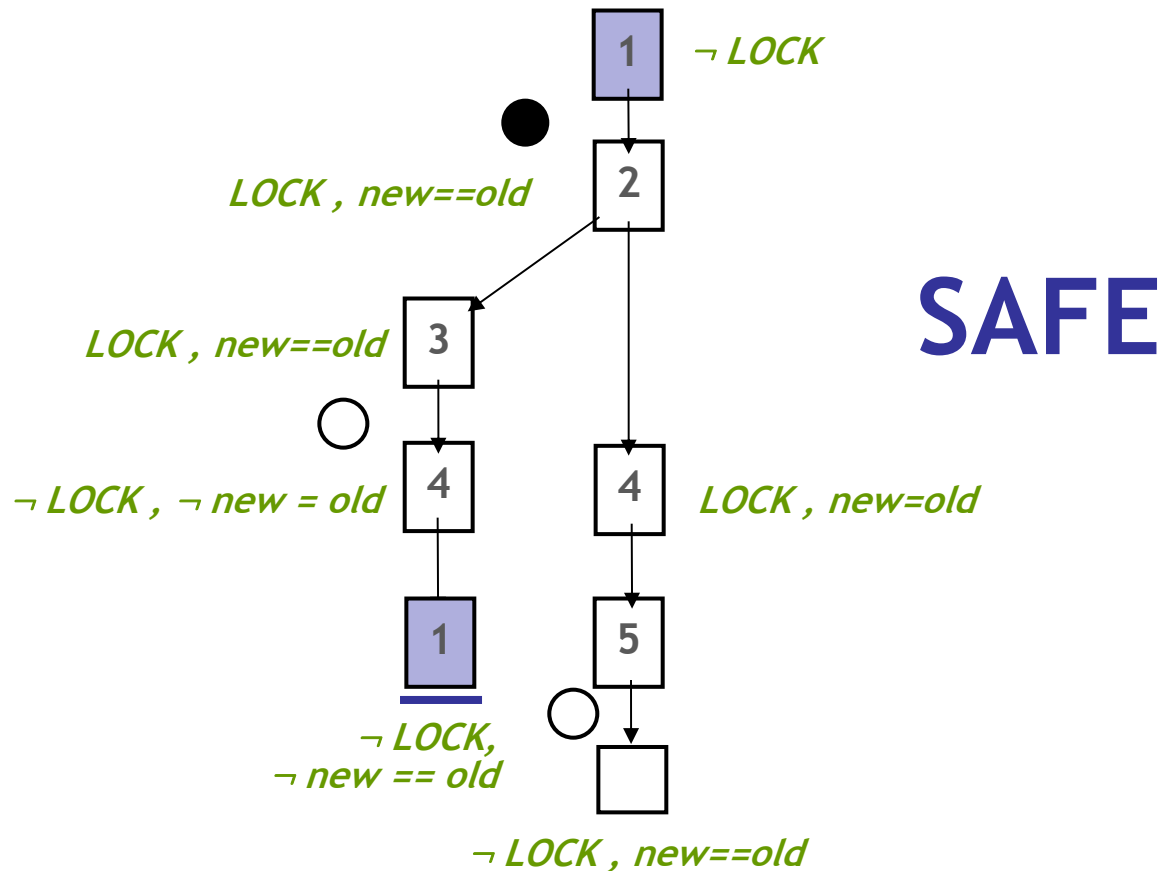
Reachability Tree

Repeat Build-and-Search

```
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1: do{  
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     unlock ();  
     new ++;  
    }  
4: }while(new != old);  
5: unlock ();  
}
```

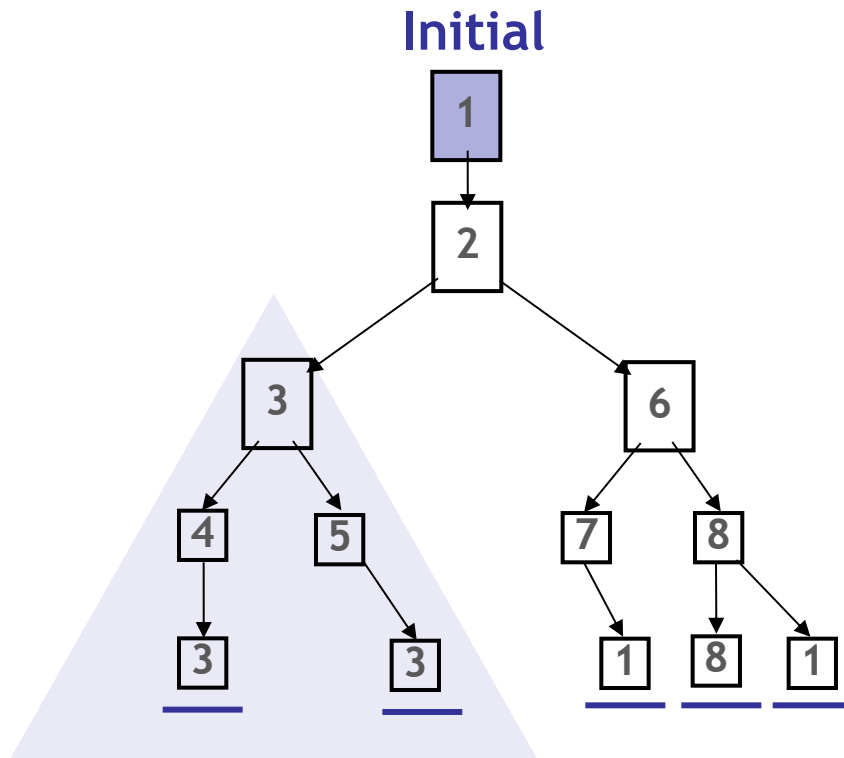


Predicates: *LOCK, new==old*



Reachability Tree

Key Idea: Reachability Tree



Unroll

1. Pick tree-node (=abs. state)
2. Add children (=abs. successors)
3. On **re-visiting** abs. state, **cut-off**

Find min spurious suffix

- Learn new predicates
- Rebuild subtree with new preds.

Error Free

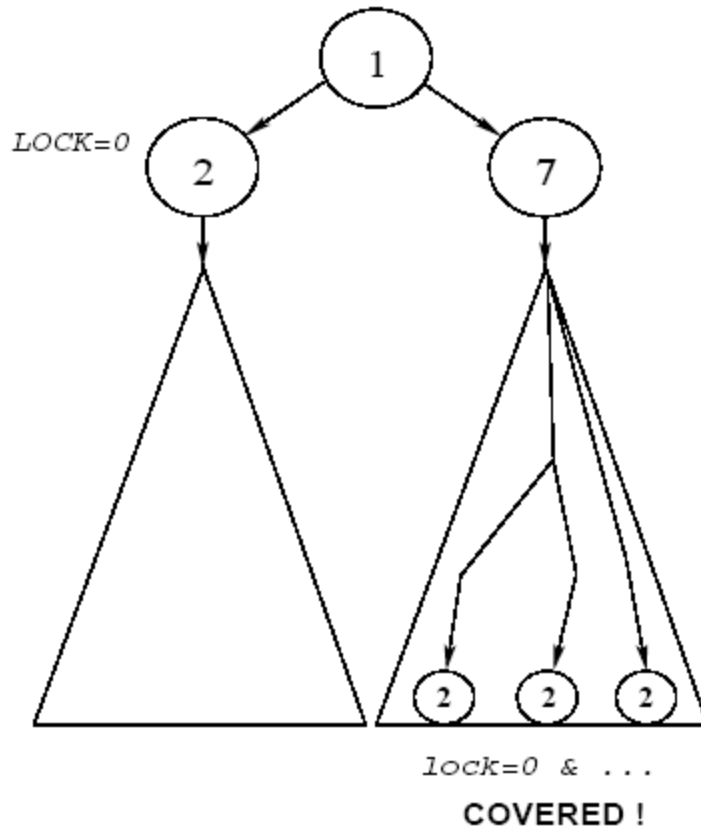
SAFE

S1: Only Abstract Reachable States

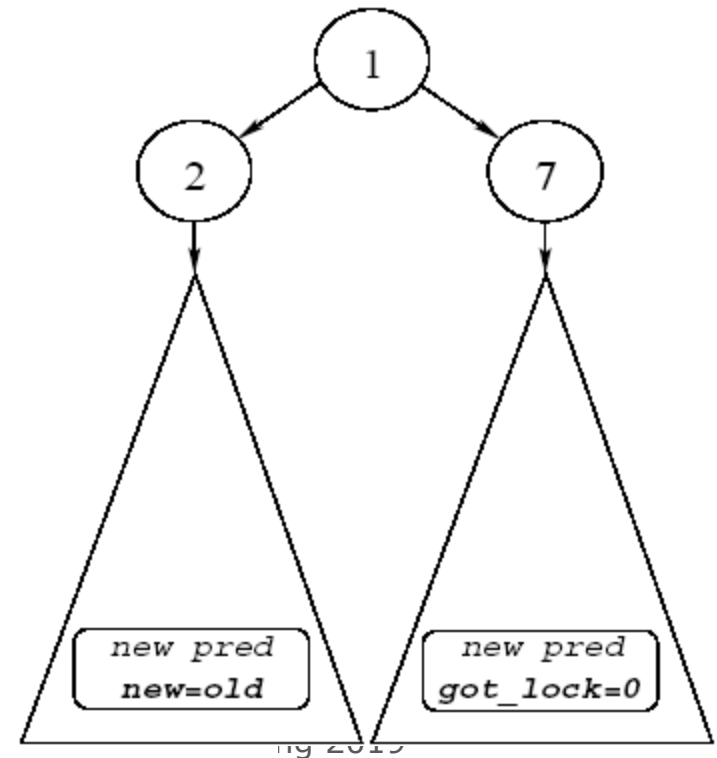
S2: Don't refine error-free regions

Blast Techniques, Graphically

- Explores reachable state, not all paths
 - Stops when state already seen on another path



- Lazy Abstraction
 - Uses predicates on demand
 - Only applies predicate to relevant part of tree



Experimental Results

Program	Postprocessed LOC	Predicates		BLAST Time (sec)	Ctrex analysis (sec)	Proof Size (bytes)
		Total	Active			
qpmouse.c	23539	2	2	0.50	0.00	175
ide.c	18131	5	5	4.59	0.01	253
aha152x.c	17736	2	2	20.93	0.00	
tlan.c	16506	5	4	428.63	403.33	405
cdaudio.c	17798	85	45	1398.62	540.96	156787
floppy.c	17386	62	37	2086.35	1565.34	
[fixed]		93	44	395.97	17.46	60129
kbfiltr.c	12131	54	40	64.16	5.89	
		48	35	256.92	165.25	
[fixed]		37	34	10.00	0.38	7619
mouclass.c	17372	57	46	54.46	3.34	
parport.c	61781	193	50	1980.09	519.69	102967

Termination

- Not guaranteed
 - The system could go on generating predicates forever
- Can guarantee termination
 - Restrict the set of possible predicates to a finite subset
 - Finite height lattices in data flow analysis!
 - Those predicates are enough to predict observable behavior of program
 - E.g. the ordering of lock and unlock statements
 - Predicates are restricted in practice
 - E.g. likely can't handle arbitrary quantification as in Dafny
 - Model checking is hard if properties depend on heap data, for example
 - Can't prove arbitrary properties in this case
- In practice
 - Terminate abstraction refinement after a time bound

Key Points of CEGAR

- To prove a property, may need to strengthen it
 - Just like strengthening induction hypothesis
- CEGAR figures out strengthening automatically
 - From analyzing why errors are spurious
- Blast uses *lazy abstraction*
 - Only uses an abstraction in the parts of the program where it is needed
 - Only builds the part of the abstract state that is reached
 - Explored state space is ***much*** smaller than potential state space

Blast in Practice

- Has scaled past 100,000 lines of code
 - Realistically starts producing worse results after a few 10K lines
- Sound up to certain limitations
 - Assumes restricted (“safe”) use of C
 - No aliases of different types; how realistic?
 - No recursion, no function pointers
 - Need models for library functions
- Has also been used to find memory safety errors, race conditions, generate test cases