Model Checking and Temporal Logics

Claire Le Goues

Incorporating slides developed by Jonathan Aldrich, which are based on slides developed by Natasha Sharygina and used and adapted by permission; as well as slides developed by Wes Weimer, also used and adapted with permission.

Model Checker: A program that checks if a (transition) system satisfies a (temporal) property.

High level definition

- **Model checking** is an automated technique that exhaustively explores the **state space** of a system, typically to see if an error state is **reachable**. It produces concrete **counter-examples**.
  - The **state explosion problem** refers to the large number of states in the model.
  - **Temporal logic** allows you to specify properties with concepts like “eventually” and “always”.

[Clarke,Emerson 81][Queille,Sifakis 82]

Explicit-state Temporal Logic Model Checking

- Domain: Continuously operating concurrent systems (e.g. operating systems, hardware controllers and network protocols)
- Ongoing, reactive semantics
  - Non-terminating, infinite computations
  - Manifest non-determinism

- Systems are modeled by **finite state machines**
- **Properties** are written in **propositional temporal logic** [Pneuli 77]
- Verification procedure is an **exhaustive search** of the **state space** of the design
- Produces diagnostic **counterexamples**.

Motivation: What can be Verified?

- Architecture
  - Will these two components interact properly?
    - Allen and Garlan: Wright system checks architectures for deadlock
- Code
  - Am I using this component correctly?
    - Microsoft's Static Driver Verifier ensures complex device driver rules are followed
      - Substantially reduced Windows blue screens
  - Is my code safe
    - Will it avoid error conditions?
    - Will it be responsive, eventually accepting the next input?
- Security
  - Is the protocol I'm using secure
    - Model checking has found defects in security protocols

Temporal Properties

- Temporal Property: A property with time-related operators such as "invariant" or "eventually"
- Invariant(p): is true in a state if property p is true in every state on all execution paths starting at that state
  - The Invariant operator has different names in different temporal logics:
    - **G, AG, $\Box$ ("goal" or "box" or "forall")**
- Eventually(p): is true in a state if property p is true at some state on every execution path starting from that state
    - **F, AF, $\Diamond$ ("diamond" or "future" or "exists")**

What is Model Checking?

Does model M satisfy a property P ?
(written M |= P)

What is "M"?

What is "P"?

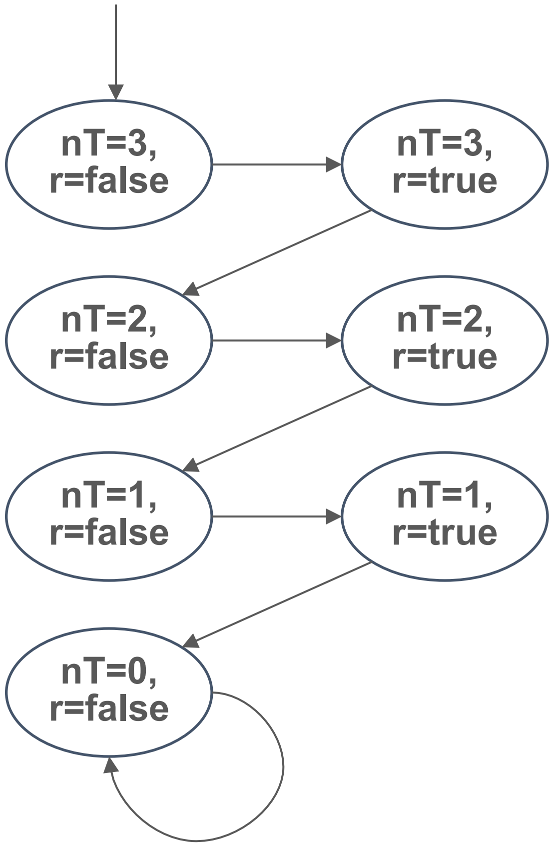
What is "satisfy"?

What is “M”?

Example Program:

```
precondition: numTickets > 0
reserved = false;
while (true) {
    getQuery();
    if (numTickets > 0 && !reserved)
        reserved = true;
    if (numTickets > 0 && reserved) {
        reserved = false;
        numTickets--;
    }
}
```

State Transition Diagram



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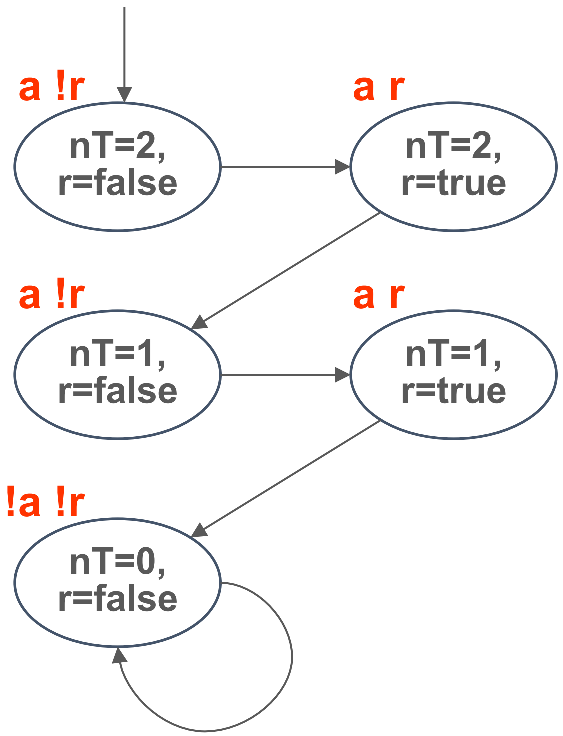
}
```

What is interesting about this?

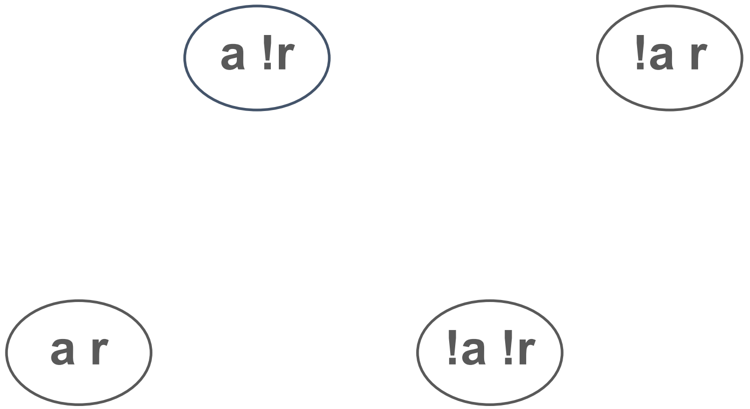
Are tickets available?	a
Is a ticket reserved?	r



What is “M”?

Abstracted Program: fewer states

```
precondition: available == true
reserved = false;
while (true) {
    getQuery();
    if (available && !reserved)
        reserved = true;
    if (available && reserved) {
        reserved = false;
        available = ?;
    }
}
```



State Transition Graph or Kripke Model

What is “M”?

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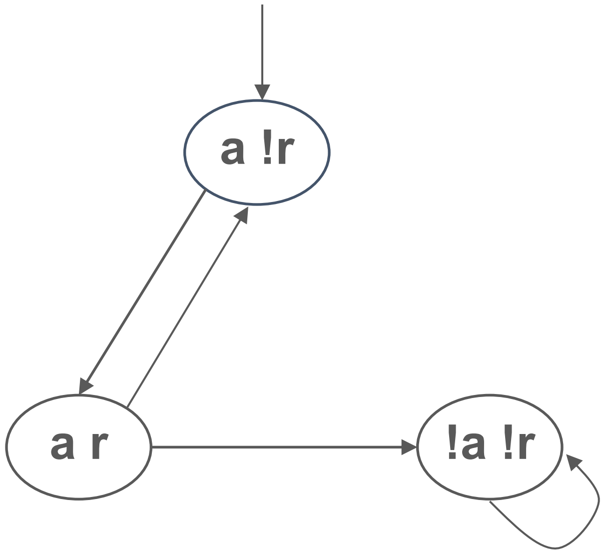


State Transition Graph or Kripke Model

What is "M"?

State: valuations to all variables

concrete state: (numTickets=5, reserved=false)

abstract state: (a=true, r=false)

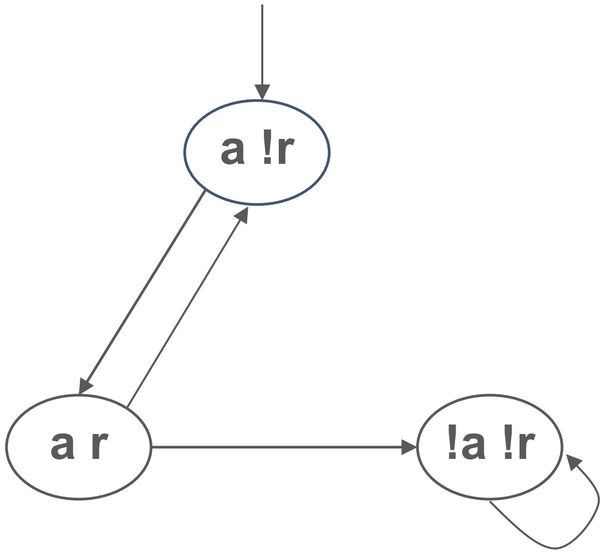
Initial states: subset of states

Arcs: transitions between states

Atomic Propositions:

a: numTickets > 0

r: reserved = true



State Transition Graph or Kripke Model

An Example Concurrent Program

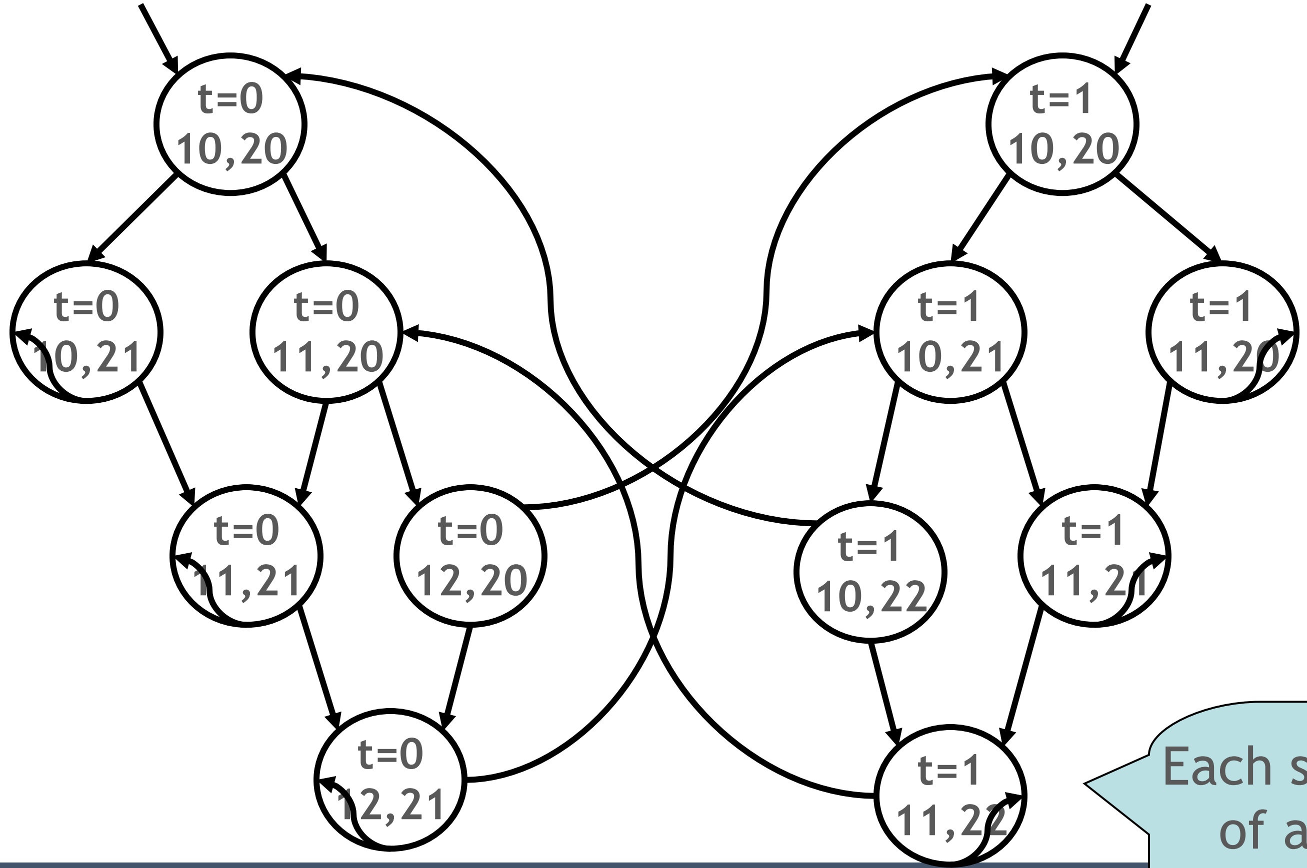
- A simple concurrent mutual exclusion program
- Two processes execute asynchronously
- There is a shared variable `turn`
- Two processes use the shared variable to ensure that they are not in the critical section at the same time
- Can be viewed as a “fundamental” program: any bigger concurrent one would include this one

```
10: while True do
11:     wait(turn = 0);
    // critical section
12:     work(); turn := 1;
13:   end while;

|| // concurrently with

20: while True do
21:    wait(turn = 1);
     // critical section
22:    work(); turn := 0;
23: end while
```

Reachable States of the Example Program

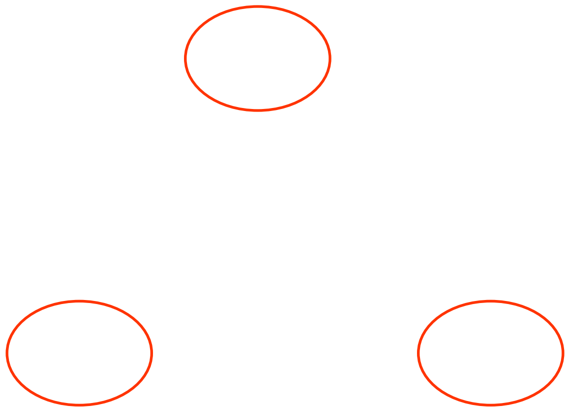


Next: formalize this intuition …

Each state is a valuation of all the variables: turn and the two program counters for two processes

What is "M"? A Labelled Transition System

$$M = \langle S, S_0, R, L \rangle$$



Kripke structure:

S – finite set of states

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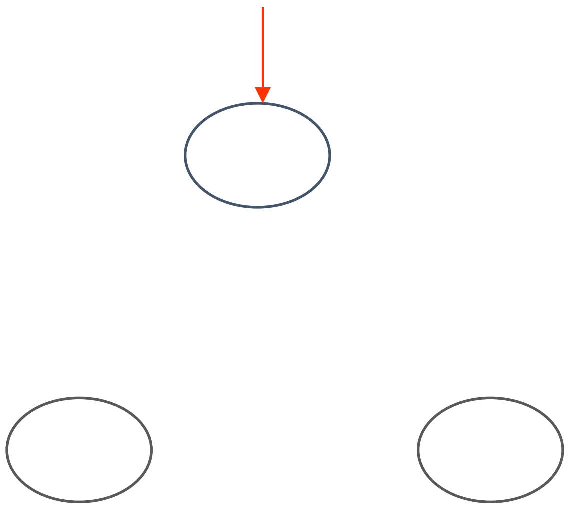

Kripke structure:

S – finite set of states

$S_0 \subseteq S$ – set of initial states

What is "M"? A Labelled Transition System

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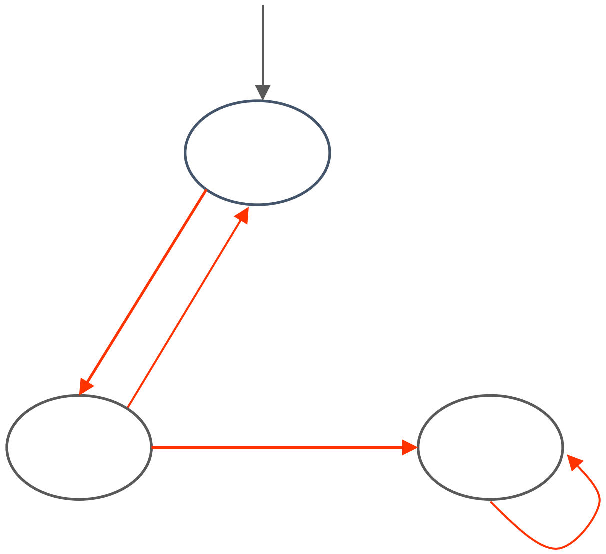
Kripke structure:

S – finite set of states

$S_0 \subseteq S$ – set of initial states

$R \subseteq S \times S$ – set of arcs



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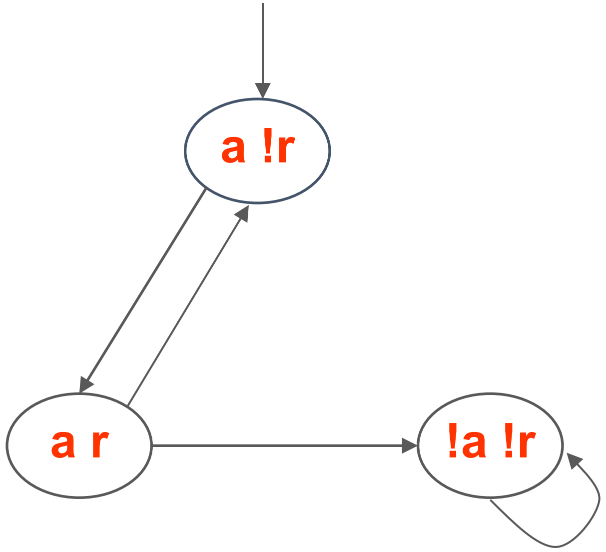


Kripke structure:

- S – finite set of states
- $S_0 \subseteq S$ – set of initial states
- $R \subseteq S \times S$ – set of arcs
- $L : S \rightarrow 2^{AP}$ – mapping from states to a set of atomic propositions

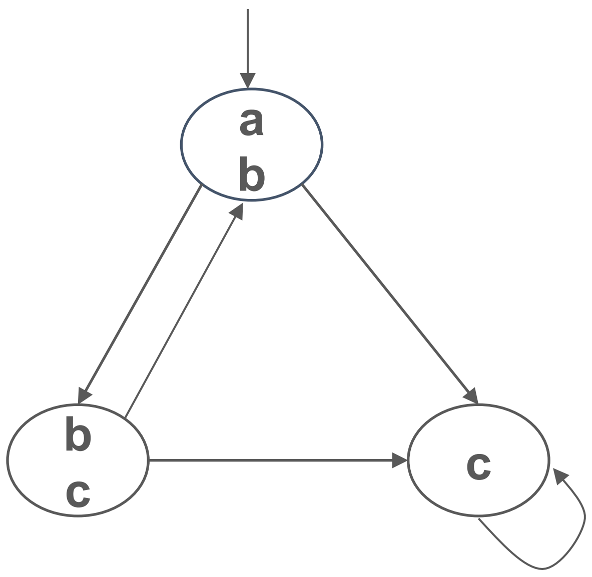
(e.g., "x=5"$\in$AP)

- Atomic propositions capture basic properties
- For software, atomic props depend on variable values
- The labeling function labels each state with the set of propositions true in that state

Atomic Propositions

- We must decide in advance which facts are important.
  - We can have “x=5” or “x=6”, but not “x”; similarly for relations (e.g., “x<y”).
- Example: “In all the reachable states (configurations) of the system, the two processes are never in the critical section at the same time”
  - Equivalently, we can say that: Invariant($\neg$(pc1=12 $\wedge$ pc2=22))
- Also: “Eventually the first process enters the critical section”
  - Eventually(pc1=12)
- “pc1=12”, “pc2=22” are atomic properties

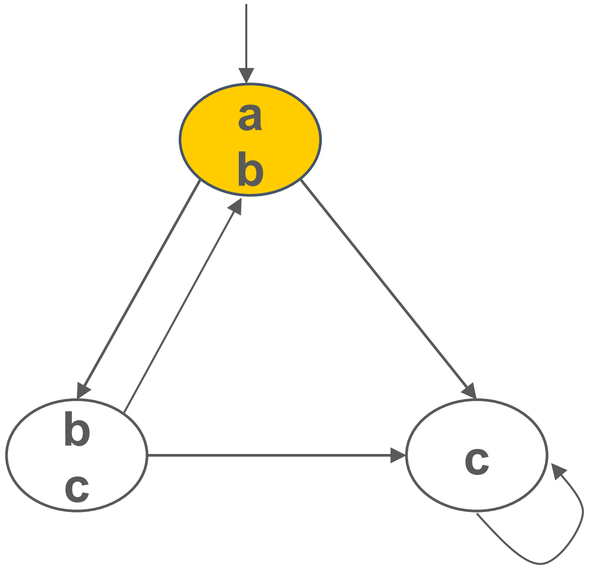
Model of Computation



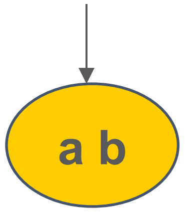
State Transition Graph **Computation Traces**

Unwind State Graph to obtain traces. *A trace* is an infinite sequence of states. The *semantics* of a FSM is a set of traces.

Model of Computation



State Transition Graph



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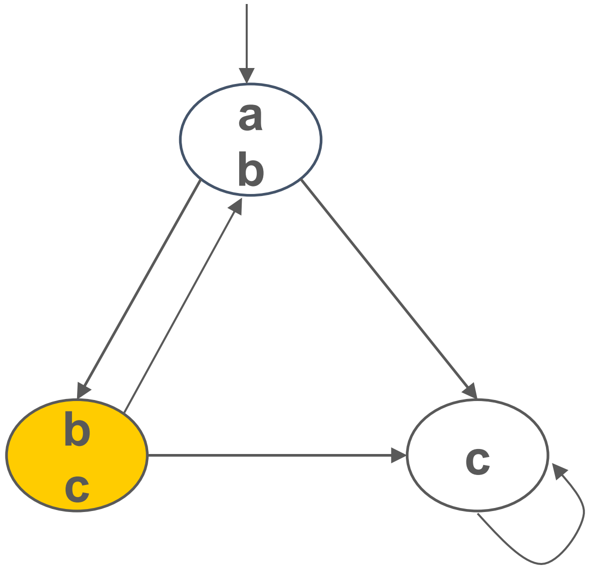

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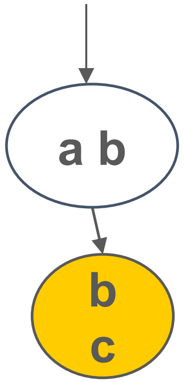

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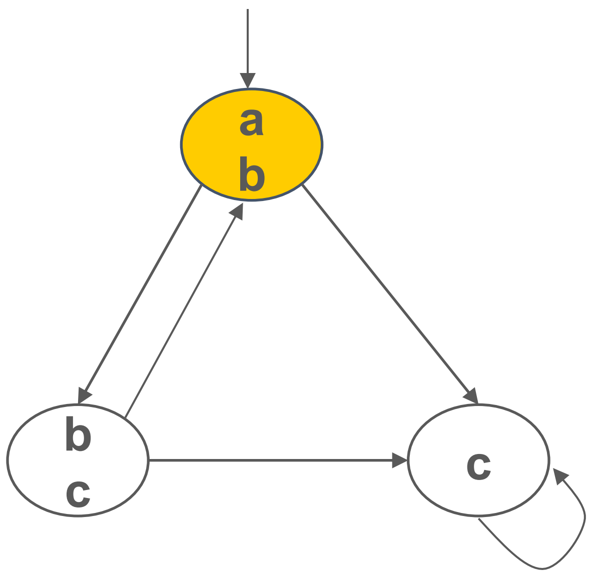

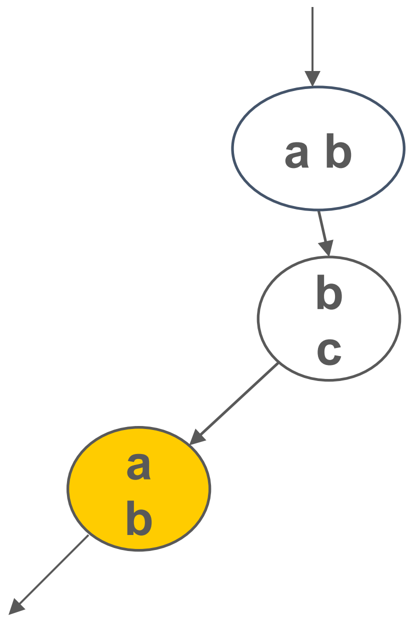

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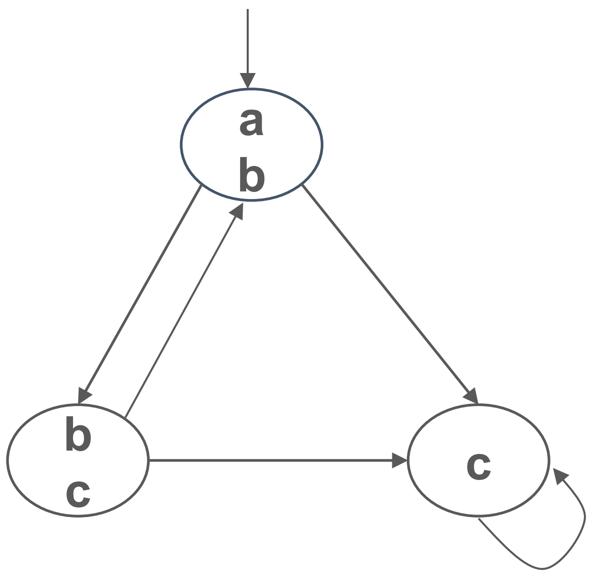
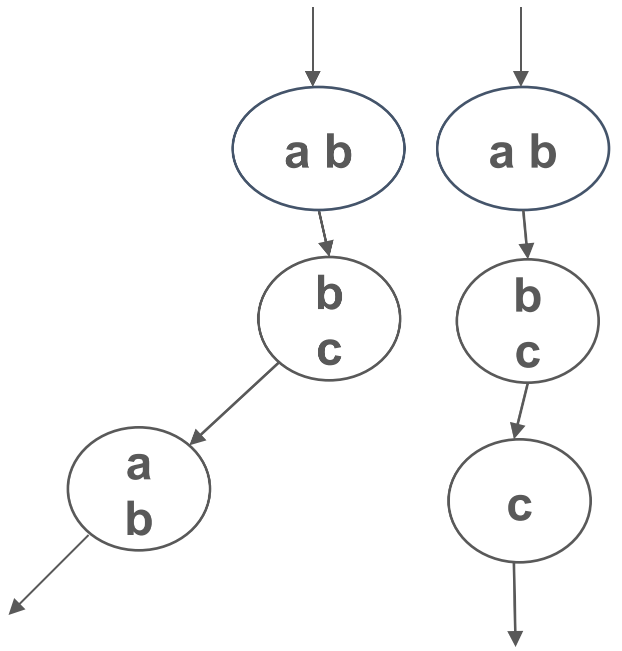


State Transition Graph



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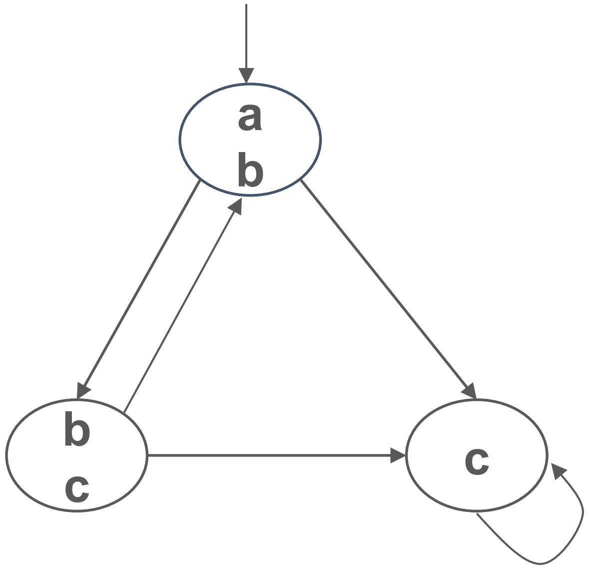

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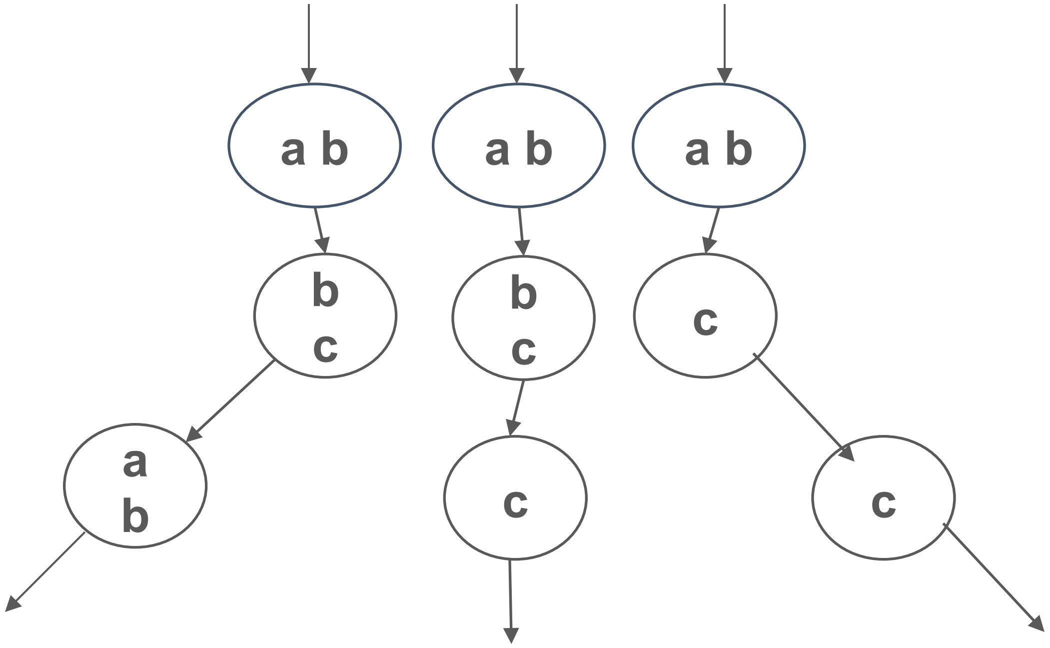

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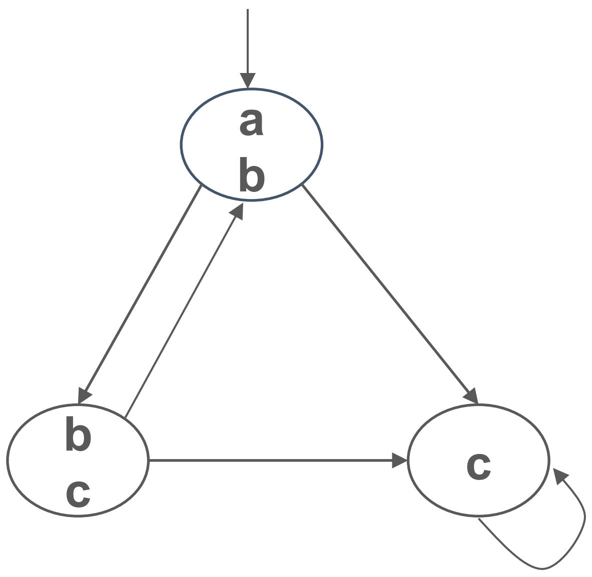

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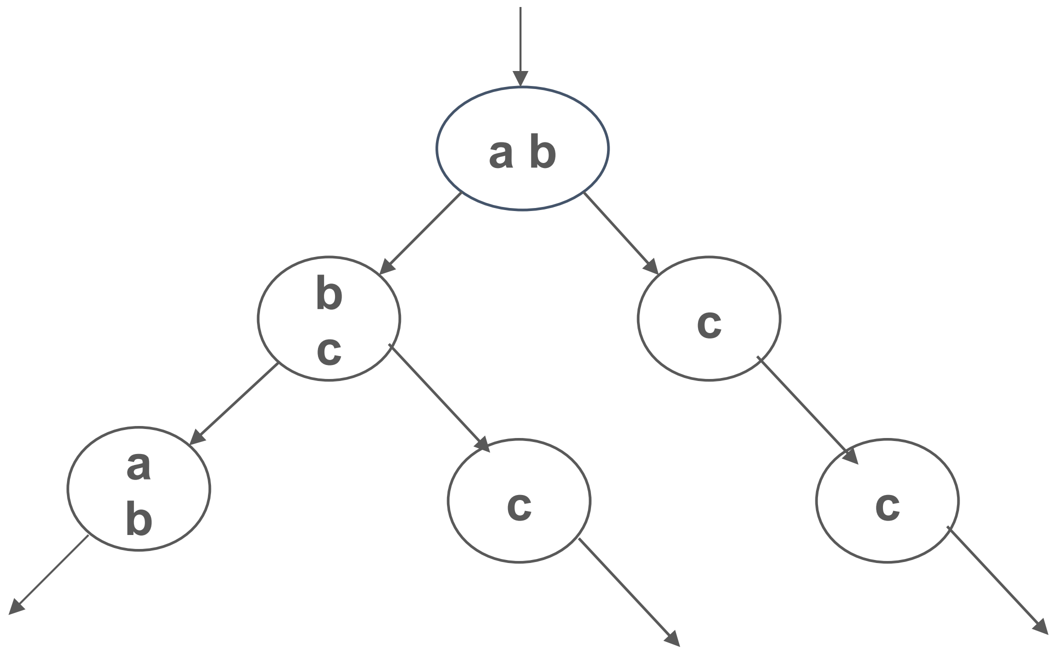


Infinite Computation Tree

Represent all traces with an infinite computation tree

What is "P"?

Different kinds of temporal logics

Syntax: What are the formulas in the logic?

Semantics: What does it mean for model **M** to satisfy formula **P**?

Formulas:

- Atomic propositions: properties of states
- Temporal Logic Specifications: properties of traces.

Computation Tree Logics

Examples: ***Safety*** (mutual exclusion): no two processes can be at a critical section at the same time

Liveness (absence of starvation): every request will be eventually granted

Temporal logics differ according to how they handle branching in the underlying computation tree.

In a linear temporal logic (LTL), operators are provided for describing system behavior along a single computation path.

In a branching-time logic (CTL), the temporal operators quantify over the paths that are possible from a given state.

Temporal Logics

- There are four basic temporal operators:
  - *X p* = Next p, p holds in the next state
  - *G p* = Globally p, p holds in every state, p is an invariant
  - *F p* = Future p, p will hold in a future state, p holds eventually
  - *p U q* = p Until q, assertion p will hold until q holds
- Precise meaning of these temporal operators are defined on execution paths

Execution Paths

- A path π in M is an infinite sequence of states (s0, s1, s2, ...), such that $\forall\, i \geq 0.\ (s_i, s_{i+1}) \in R$
  - π^i denotes the suffix of π starting at s_i
- M, $\pi \vDash f$ means that f holds along path π in the Kripke structure M,
  - “the path π in the transition system makes the temporal logic predicate f true”
  - Example: $A\ \pi.\ \pi \vDash G\ (\neg(pc1{=}12 \wedge pc2{=}22))$
- In some temporal logics one can quantify the paths starting from a state using path quantifiers
  - A : for all paths
  - E : there exists a path

Summary: Formulas over States and Paths

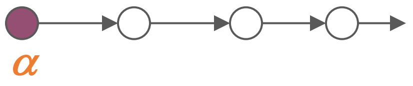
- State formulas
  - Describe a property of a state in a model M
  - If $p \in AP$, then p is a state formula
  - If f and g are state formulas, then $\neg f$, $f \wedge g$ and $f \vee g$ are state formulas
  - If f is a path formula, then **E** f and **A** f are state formulas
- Path formulas
  - Describe a property of an infinite path through a model M
  - If f is a state formula, then f is also a path formula
  - If f and g are path formulas, then $\neg f$, $f \wedge g$, $f \vee g$, **X** f, **F** f, **G** f, and f **U** g are path formulas

LTL logic operators wrt Paths

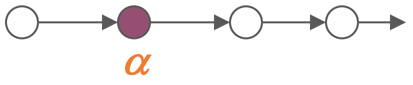
Linear Time Logic (LTL) [Pnueli 77]: logic of temporal sequences.

- LTL properties are constructed from atomic propositions in AP; logical operators $\wedge$, $\vee$, $\neg$; and temporal operators X, G, F, U.
- The semantics of LTL properties is defined on paths:

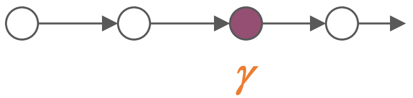
- α: α **holds in the current state (atomic)**



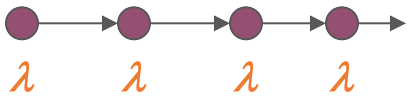
- **X**α: α **holds in the next state (Next)**



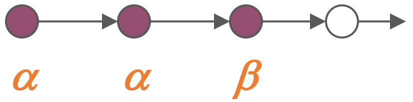
- **F**γ: γ **holds eventually (Future)**



- **G**λ: λ **holds from now on (Globally)**



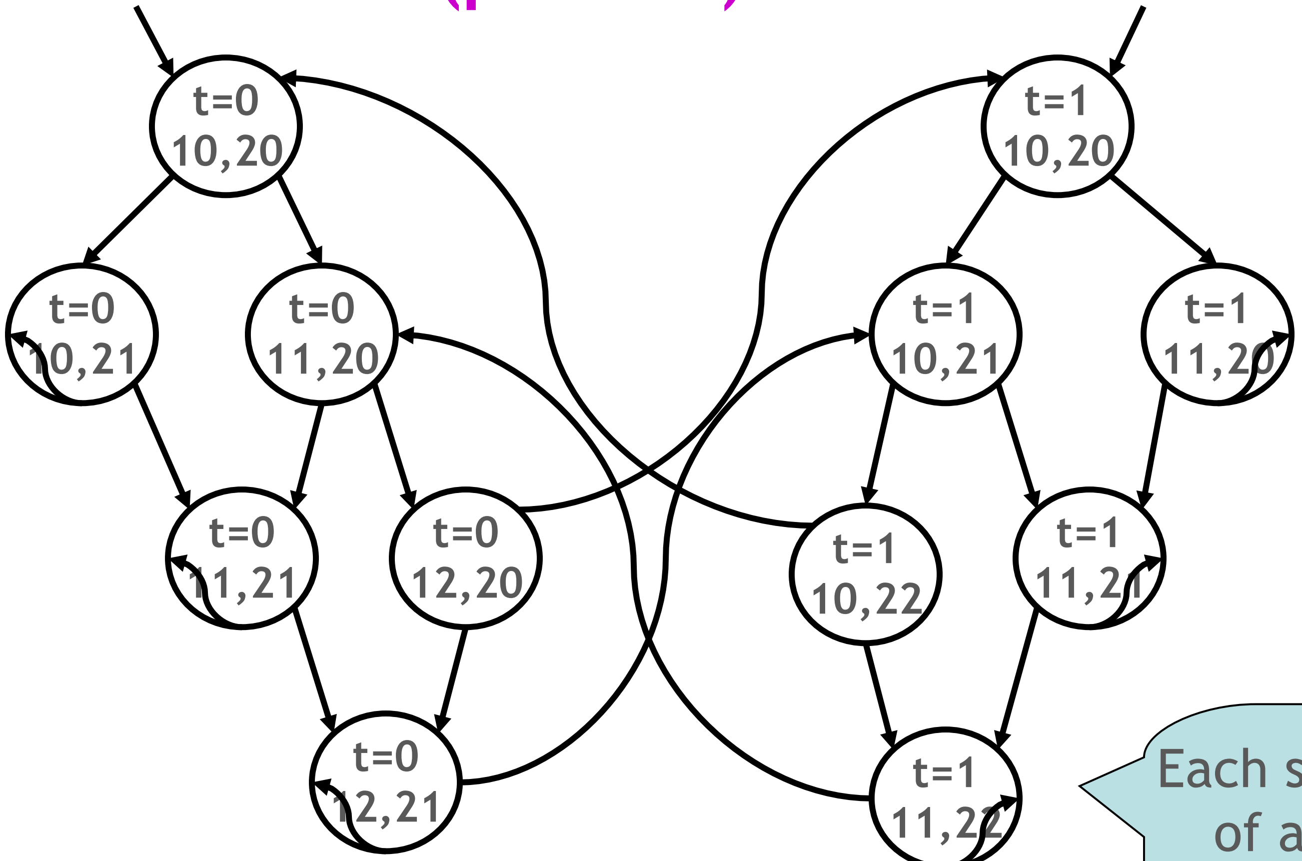
- (α **U** β): α **holds until** β **holds (Until)**



Satisfying Linear Time Logic

- Given a transition system T = (S, I, R, L) and an LTL property p, T satisfies p if all paths starting from all initial states I satisfy p
- Example LTL formulas:
  - *Invariant*($\neg$(pc1=12 $\wedge$ pc2=22)):
    G($\neg$(pc1=12 $\wedge$ pc2=22))
  - *Eventually*(pc1=12):
    F(pc1=12)

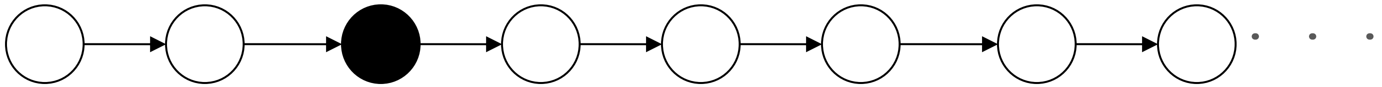
- *Invariant*(¬(pc1=12 ∧ pc2=22)):
  G(¬(pc1=12 ∧ pc2=22))
- *Eventually*(pc1=12):
  F(pc1=12)



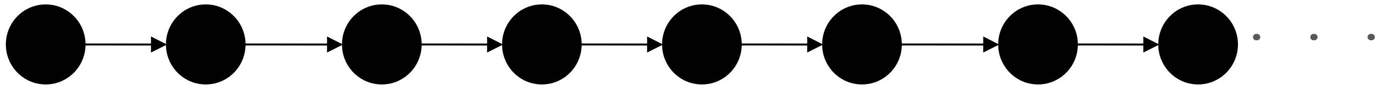
Each state is a valuation of all the variables: turn and the two program counters for two processes

LTL Satisfiability Examples

p does not hold   p holds



On this path: F p holds, G p does not hold, p does not hold, X p does not hold, X (X p) holds, X (X (X p)) does not hold



On this path: F p holds, G p holds, p holds, X p holds, X (X p) holds, X (X (X p))) holds

Typical LTL Formulas

- **G** (*Req* $\Rightarrow$ **F** *Ack*): whenever *Request* occurs, it will be eventually *Acknowledged.*
- **G** (*DeviceEnabled*): *DeviceEnabled* always holds on every computation path.
- **G** (**F** *Restart*): Fairness: from any state one will eventually get to a *Restart* state. I.e. *Restart* states occur infinitely often.
- **G** (*Reset* $\Rightarrow$ **F** *Restart*): whenever the reset button is pressed one will eventually get to the *Restart* state.
- Pedantic note:
  - G is sometimes written □
  - F is sometimes written ◇

Practice Writing Properties

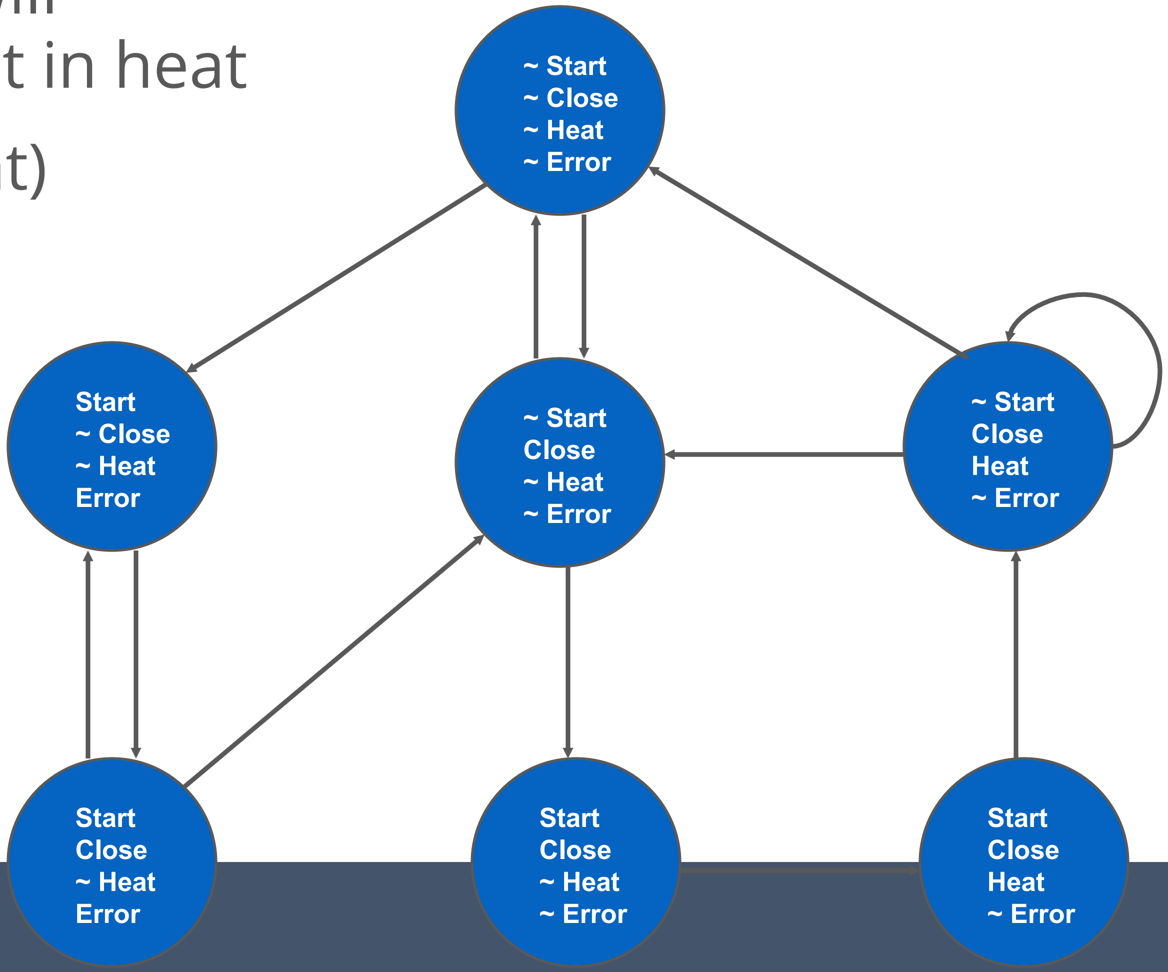
- If the door is locked, it will not open until someone unlocks it
  - assume atomic predicates locked, unlocked, open
- If you press ctrl-C, you will get a command line prompt
- The saw will not run unless the safety guard is engaged

Practice Writing Properties

- If the door is locked, it will not open until someone unlocks it
  - assume atomic predicates locked, unlocked, open
  - $G\ (locked \Rightarrow (\neg open\ U\ unlocked))$
- If you press ctrl-C, you will get a command line prompt
  - $G\ (ctrlC \Rightarrow F\ prompt)$
- The saw will not run unless the safety guard is engaged
  - $G\ (\neg safety \Rightarrow \neg running)$

LTL Model Checking Example

- Pressing Start will eventually result in heat
- $\mathbf{G}(\text{Start} \Rightarrow \mathbf{F}\ \text{Heat})$

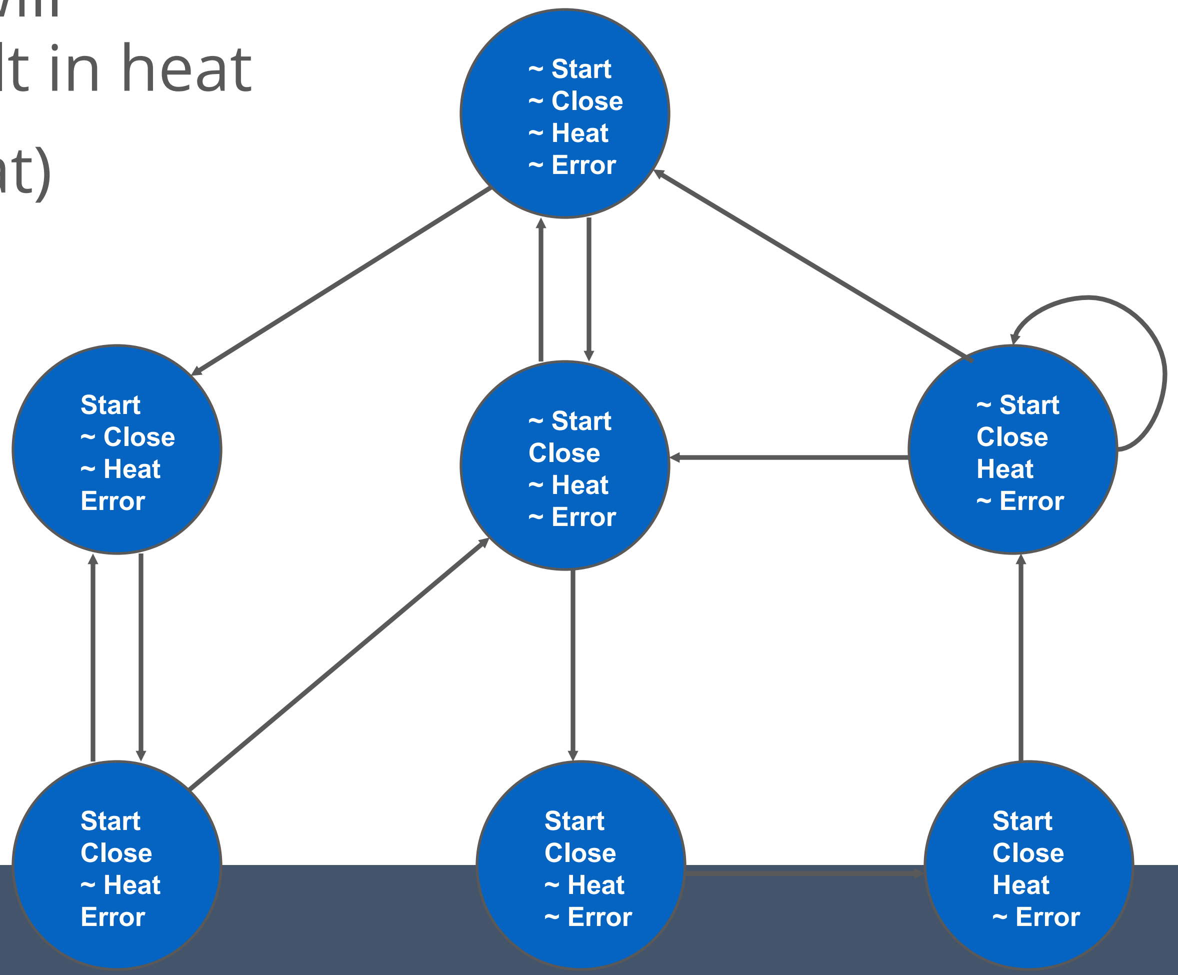


LTL Model Checking

- f (primitive formula)
  - Just check the properties of the current state
- X f
  - Verify f holds in all successors of the current state
- G f
  - Find all reachable states from the current state, and ensure f holds in all of them
    - use depth-first or breadth-first search
- f U g
  - Do a depth-first search from the current state. Stop when you get to a g or you loop back on an already visited state. Signal an error if you hit a state where f is false before you stop.
- F f
  - Harder. Intuition: look for a path from the current state that loops back on itself, such that f is false on every state in the path. If no such path is found, the formula is true.
    - Reality: use Büchi automata

LTL Model Checking Example

- Pressing Start will eventually result in heat
- $\mathbf{G}(\text{Start} \Rightarrow \mathbf{F}\ \text{Heat})$



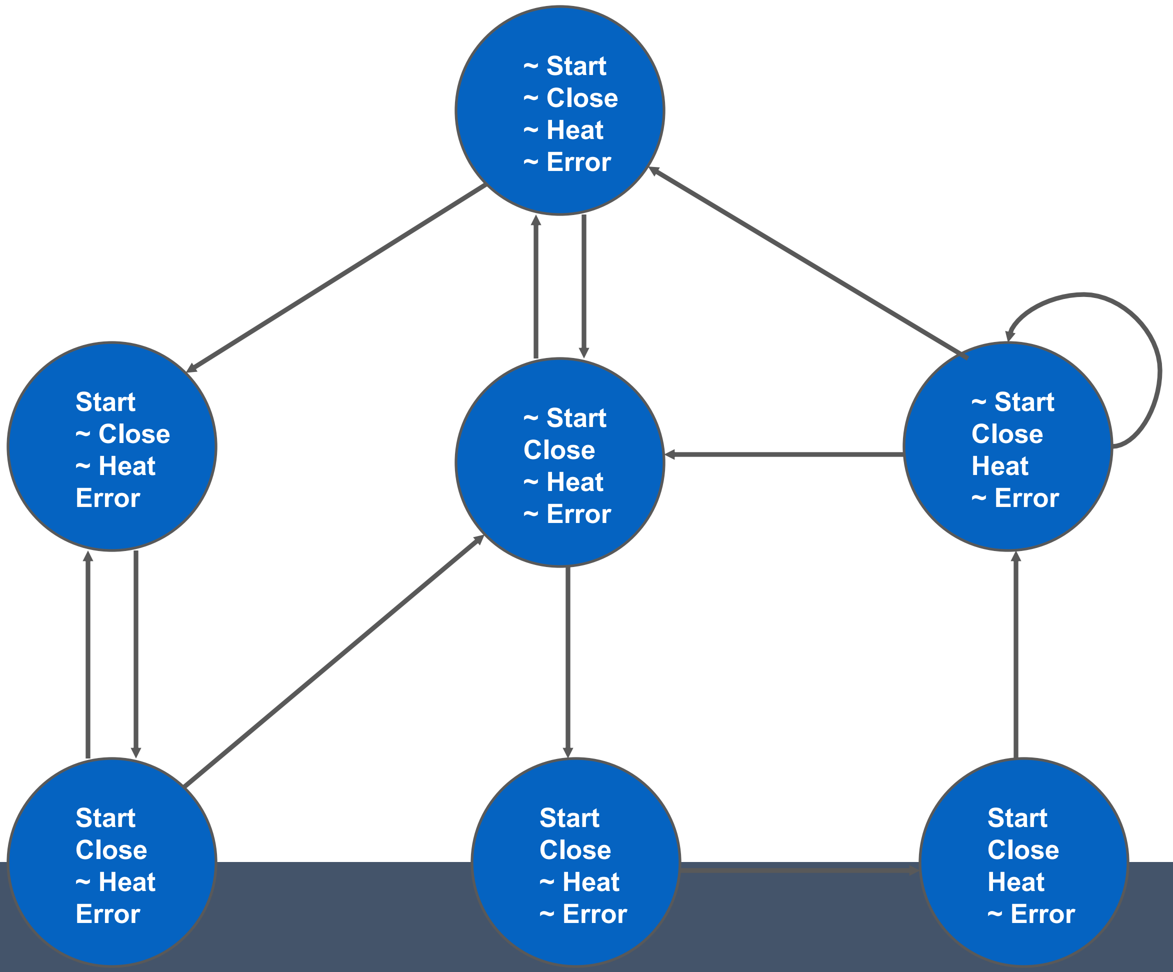
LTL Model Checking Example

- The oven doesn't heat up until the door is closed.

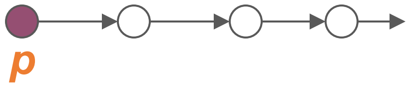
(¬Heat) **U** Close

(¬Heat) **W** Close

G (not Closed => not Heat)



Semantics of LTL Formulas



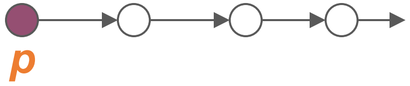
$$M, \pi \vDash p \quad \Leftrightarrow \quad \pi = s\ldots \wedge p \in L(s)$$

$$M, \pi \vDash \neg g \quad \Leftrightarrow \quad M, \pi \nvDash g$$

$$M, \pi \vDash g_1 \wedge g_2 \quad \Leftrightarrow \quad M, \pi \vDash g_1 \wedge M, \pi \vDash g_2$$

$$M, \pi \vDash g_1 \vee g_2 \quad \Leftrightarrow \quad M, \pi \vDash g_1 \vee M, \pi \vDash g_2$$

Semantics of LTL Formulas

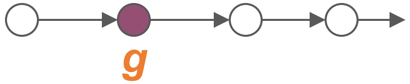


$$M, \pi \vDash p \quad \Leftrightarrow \quad \pi = s\ldots \wedge p \in L(s)$$

$$M, \pi \vDash \neg g \quad \Leftrightarrow \quad M, \pi \nvDash g$$

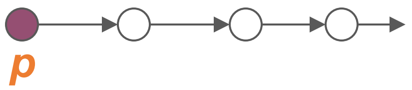
$$M, \pi \vDash g_1 \wedge g_2 \quad \Leftrightarrow \quad M, \pi \vDash g_1 \wedge M, \pi \vDash g_2$$

$$M, \pi \vDash g_1 \vee g_2 \quad \Leftrightarrow \quad M, \pi \vDash g_1 \vee M, \pi \vDash g_2$$



$$M, \pi \vDash \mathbf{X}\, g \quad \Leftrightarrow \quad M, \pi^1 \vDash g$$

Semantics of LTL Formulas



$$M, \pi \vDash p \quad \Leftrightarrow \quad \pi = s\ldots \wedge p \in L(s)$$

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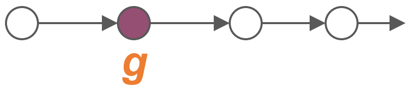
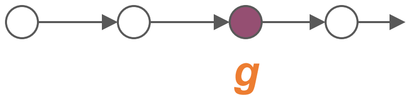
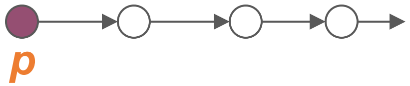


$$M, \pi \vDash \mathbf{X}\, g \quad \Leftrightarrow \quad M, \pi^1 \vDash g$$



$$M, \pi \vDash \mathbf{F}\, g \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \vDash g$$

Semantics of LTL Formulas



$$M, \pi \vDash p \quad \Leftrightarrow \quad \pi = s\ldots \wedge p \in L(s)$$

$$M, \pi \vDash \neg g \quad \Leftrightarrow \quad M, \pi \nvDash g$$

$$M, \pi \vDash g_1 \wedge g_2 \quad \Leftrightarrow \quad M, \pi \vDash g_1 \wedge M, \pi \vDash g_2$$

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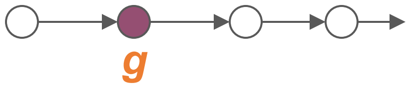
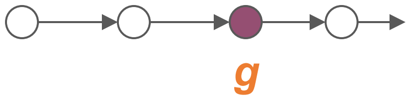
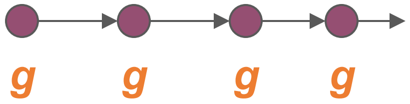


$$M, \pi \vDash \mathbf{X}\, g \quad \Leftrightarrow \quad M, \pi^1 \vDash g$$

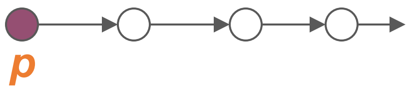


$$M, \pi \vDash \mathbf{F}\, g \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \vDash g$$



$$M, \pi \vDash \mathbf{G}\, g \quad \Leftrightarrow \quad \forall k \geq 0 \mid M, \pi^k \vDash g$$

Semantics of LTL Formulas



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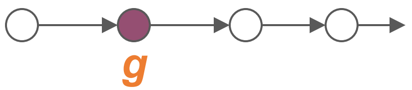
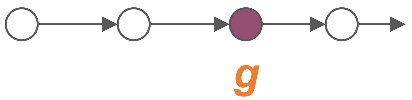
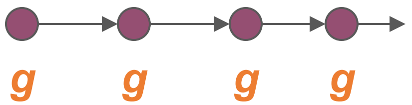


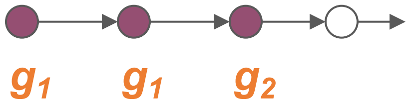
$$M, \pi \vDash \mathbf{X}\, g \quad \Leftrightarrow \quad M, \pi^1 \vDash g$$



$$M, \pi \vDash \mathbf{F}\, g \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \vDash g$$

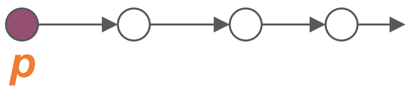


$$M, \pi \vDash \mathbf{G}\, g \quad \Leftrightarrow \quad \forall k \geq 0 \mid M, \pi^k \vDash g$$



$$M, \pi \vDash g_1\, \mathbf{U}\, g_2 \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \vDash g_2 \wedge \forall 0 \leq j < k\; M, \pi^j \vDash g_1$$

Semantics of LTL Formulas

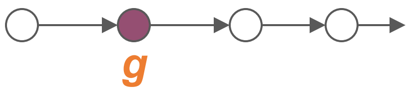


$M, \pi \models p \quad \Leftrightarrow \quad \pi = s\ldots \wedge p \in L(s)$

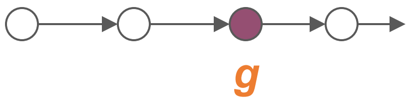
$M, \pi \models \neg g \quad \Leftrightarrow \quad M, \pi \not\models g$

$M, \pi \models g_1 \wedge g_2 \quad \Leftrightarrow \quad M, \pi \models g_1 \wedge M, \pi \models g_2$

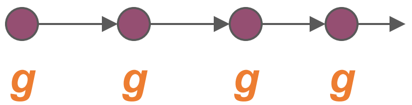
$M, \pi \models g_1 \vee g_2 \quad \Leftrightarrow \quad M, \pi \models g_1 \vee M, \pi \models g_2$



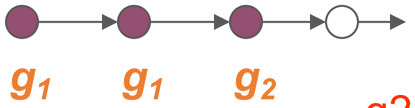
$M, \pi \models \mathbf{X}\, g \quad \Leftrightarrow \quad M, \pi^1 \models g$



$M, \pi \models \mathbf{F}\, g \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \models g$



$M, \pi \models \mathbf{G}\, g \quad \Leftrightarrow \quad \forall k \geq 0 \mid M, \pi^k \models g$



$M, \pi \models g_1\, \mathbf{U}\, g_2 \quad \Leftrightarrow \quad \exists k \geq 0 \mid M, \pi^k \models g_2 \wedge \forall 0 \leq j < k\ M, \pi^j \models g_1$

g2 must eventually hold

semantics of “until” in English are potentially unclear—that’s why we have a formal definition

Semantics of Formulas

$M, s \models p$	$\Leftrightarrow p \in L(s)$	$M, \pi \models f$	$\Leftrightarrow \pi{=}s\ldots \wedge M, s \models f$
$M, s \models \neg f$	$\Leftrightarrow M, s \not\models f$	$M, \pi \models \neg g$	$\Leftrightarrow M, \pi \not\models g$
$M, s \models f_1 \wedge f_2$	$\Leftrightarrow M, s \models f_1 \wedge M, s \models f_2$	$M, \pi \models g_1 \wedge g_2$	$\Leftrightarrow M, \pi \models g_1 \wedge M, \pi \models g_2$
$M, s \models f_1 \vee f_2$	$\Leftrightarrow M, s \models f_1 \vee M, s \models f_2$	$M, \pi \models g_1 \vee g_2$	$\Leftrightarrow M, \pi \models g_1 \vee M, \pi \models g_2$
$M, s \models \mathbf{E}\, g_1$	$\Leftrightarrow \exists \pi{=}s\ldots \mid M, \pi \models g_1$	$M, \pi \models \mathbf{X}\, g$	$\Leftrightarrow M, \pi^1 \models g$
$M, s \models \mathbf{A}\, g_1$	$\Leftrightarrow \forall \pi{=}s\ldots\ M, \pi \models g_1$	$M, \pi \models \mathbf{F}\, g$	$\Leftrightarrow \exists k \geq 0 \mid M, \pi^k \models g$
		$M, \pi \models \mathbf{G}\, g$	$\Leftrightarrow \forall k \geq 0 \mid M, \pi^k \models g$
		$M, \pi \models g_1\, \mathbf{U}\, g_2$	$\Leftrightarrow \exists k \geq 0 \mid M, \pi^k \models g_2$
			$\wedge\ \forall 0 \leq j < k\ M, \pi^j \models g_1$

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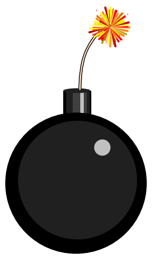
Model Checking Complexity

- Given a transition system T = (S, I, R, L) and an LTL formula f
  - One can check if the transition system satisfies the temporal logic formula f in $O(2^{|f|} \times (|S| + |R|))$ time
- Given a transition system T = (S, I, R, L) and a CTL formula f
  - One can check if a state of the transition system satisfies the temporal logic formula f in $O(|f| \times (|S| + |R|))$ time
- Model checking procedures can generate counter-examples without increasing the complexity of verification (= "for free")

State Space Explosion

Problem:

Size of the state graph can be exponential in size of the program (both in the number of the program *variables* and the number of program *components or processes*)



$$M = M_1 \mid\mid \ldots \mid\mid M_n$$

If each M_i has just 2 local states, potentially 2^n global states

Research Directions: State space reduction

Explicit-State Model Checking

- One can show the complexity results using depth first search algorithms
  - The transition system is a directed graph
  - CTL model checking is multiple depth first searches (one for each temporal operator)
  - LTL model checking is one nested depth first search (i.e., two interleaved depth-first-searches)

- Such algorithms are called explicit-state model checking algorithms.

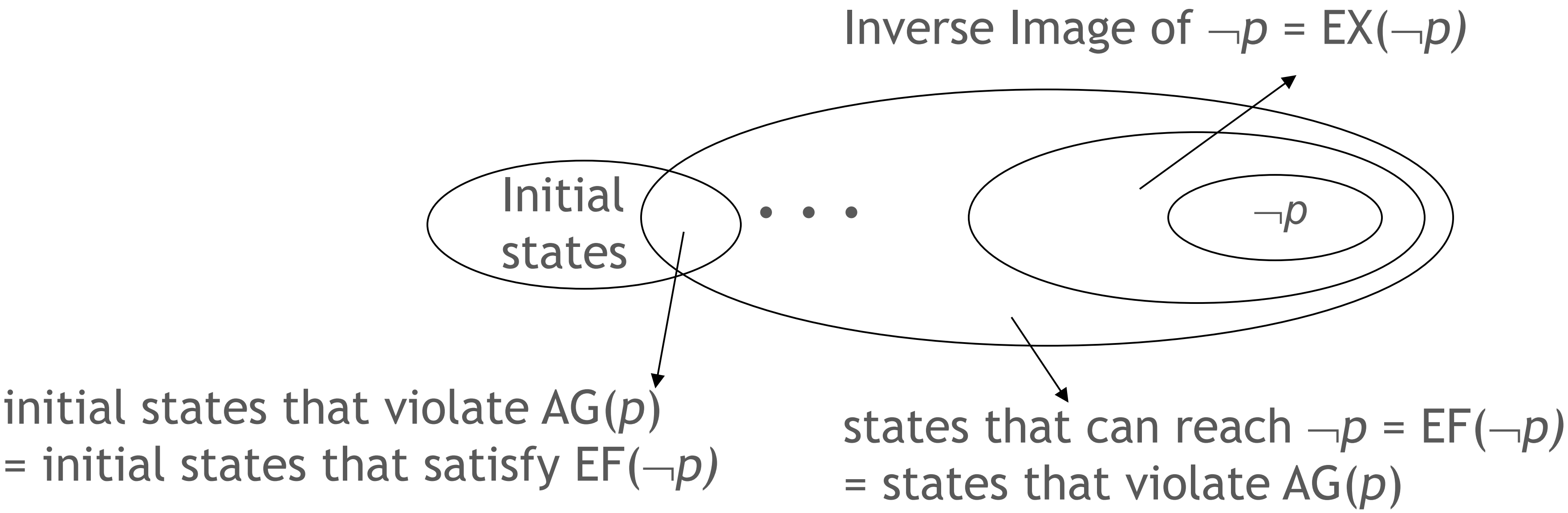
Temporal Properties ≡ Fixpoints

- States that satisfy AG(p) are all the states which are *not* in EF(¬p) (= the states that can reach ¬p)
- Compute EF(¬p) as the **fixpoint** of Func: $2^S \to 2^S$
- Given $Z \subseteq S$,
  - Func(Z) = ¬p ∪ **reach-in-one-step**(Z)
  - or Func(Z) = ¬p ∪ EX(Z)

This is called the ***inverse image*** *of Z*

- Actually, EF(¬p) is the ***least*-fixpoint** of Func
  - smallest set Z such that Z = Func(Z)
  - to compute the least fixpoint, start the iteration from Z=∅, and apply the Func until you reach a fixpoint
  - This can be computed (unlike most other fixpoints)

Pictoral Backward Fixpoint



This fixpoint computation can be used for:

- verification of EF(¬p)
- or falsification of AG(p)

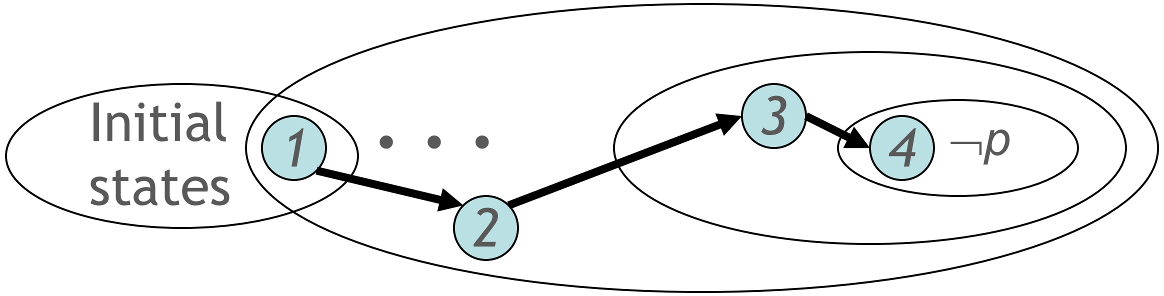
… *and a similar forward fixpoint handles the other cases*

Symbolic Model Checking

- Symbolic Model Checking represent state sets and the transition relation as *Boolean logic formulas*
  - Fixpoint computations manipulate **sets of states** rather than individual states
  - Recall: we needed to compute EX(Z), but $Z \subseteq S$
- Forward and backward fixpoints can be computed by iteratively manipulating these formulas
  - Forward, inverse image: Existential variable elimination
  - Conjunction (intersection), disjunction (union) and negation (set difference), and equivalence check
- Use an efficient data structure for manipulation of Boolean logic formulas: **Binary Decision Diagrams (BDDs)**

To produce the explicit counter-example, use the "onion-ring method"

- A counter-example is a valid execution path
- For each Image Ring (= set of states), find a state and link it with the concrete transition relation R
- Since each Ring is "reached in one step from previous ring" (e.g., Ring#3 = EX(Ring#4)) this works
- Each state z comes with L(z) so you know what is true at each point (= what the values of variables are)



Model Checking Performance/Examples

- Performance:
  - Model Checkers today can routinely handle systems with between 100 and 300 state variables.
  - Systems with 10120 reachable states have been checked.
  - By using appropriate abstraction techniques, systems with an essentially **unlimited number of states** can be checked.
- Notable examples:
  - **IEEE Scalable Coherent Interface** – In 1992 Dill's group at Stanford used Murphi to find several errors, ranging from uninitialized variables to subtle logical errors
  - **IEEE Futurebus** – In 1992 Clarke's group at CMU found previously undetected design errors
  - **PowerScale multiprocessor** (processor, memory controller, and bus arbiter) was verified by Verimag researchers using CAESAR toolbox
  - **Lucent telecom**. protocols were verified by FormalCheck – errors leading to lost transitions were identified
  - **PowerPC 620 Microprocessor** was verified by Motorola's Verdict model checker.

Efficient Algorithms for LTL Model Checking

- Use Büchi automata
  - Beyond the scope of this course

- Canonical reference on Model Checking:
  - Edmund Clarke, Orna Grumberg, and Doron A. Peled. Model Checking. MIT Press, 1999.

Computation Tree Logics

- Formulas are constructed from ***path quantifiers*** and ***temporal operators:***

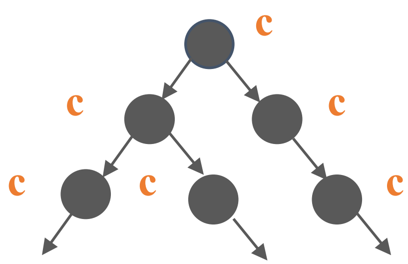
1. ***Path Quantifiers:***
    - **A** – "for every path"
    - **E** – "there exists a path"

LTL: start with an A and then use only Temporal Operators

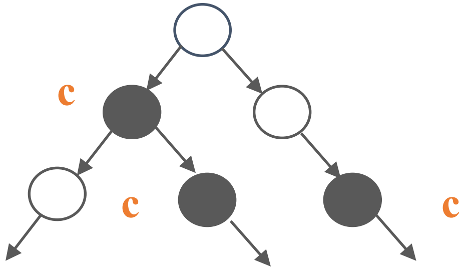
2. ***Temporal Operator:***
    - **X**α **-** α holds **next** time
    - **F**α **-** α holds sometime in the **future**
    - **G**α **-** α holds **globally** in the **future**
    - α **U**β **-** α holds **until** β holds

The Logic CTL

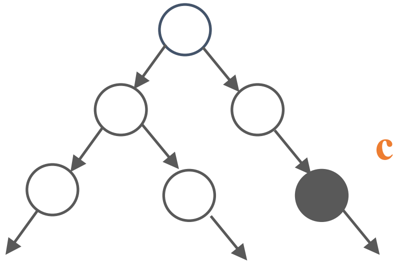
In a branching-time logic (CTL), the temporal operators quantify over the paths that are possible from a given state (s_0). Requires each temporal operator (**X**, **F**, **G**, and **U**) to be preceded by a path quantifier (**A** or **E**).



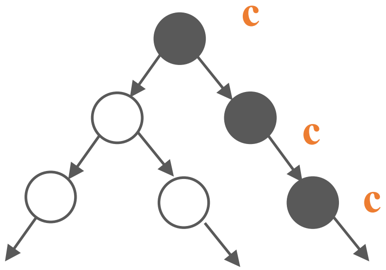
$M, s_0 \models \mathbf{AG}\ c$



$M, s_0 \models \mathbf{AF}\ c$

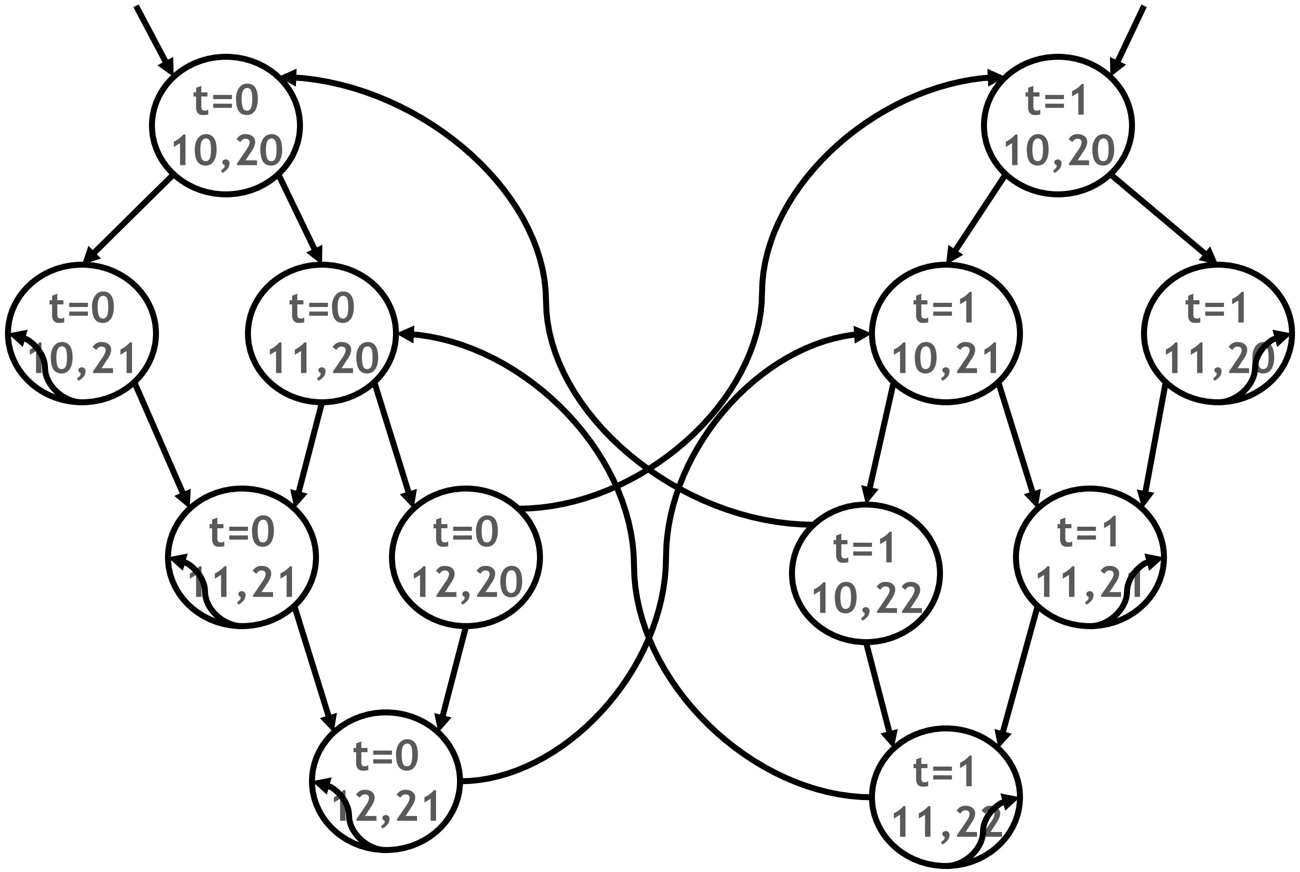


$M, s_0 \models \mathbf{EF}\ c$



$M, s_0 \models \mathbf{EG}\ c$

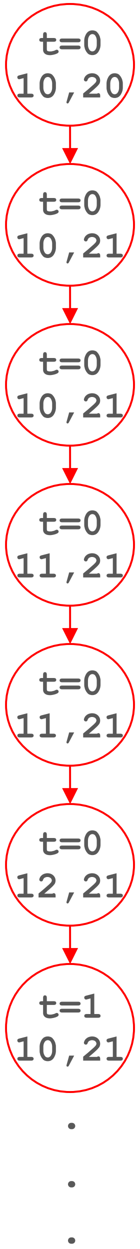
Remember the Example



Linear vs. Branching Time

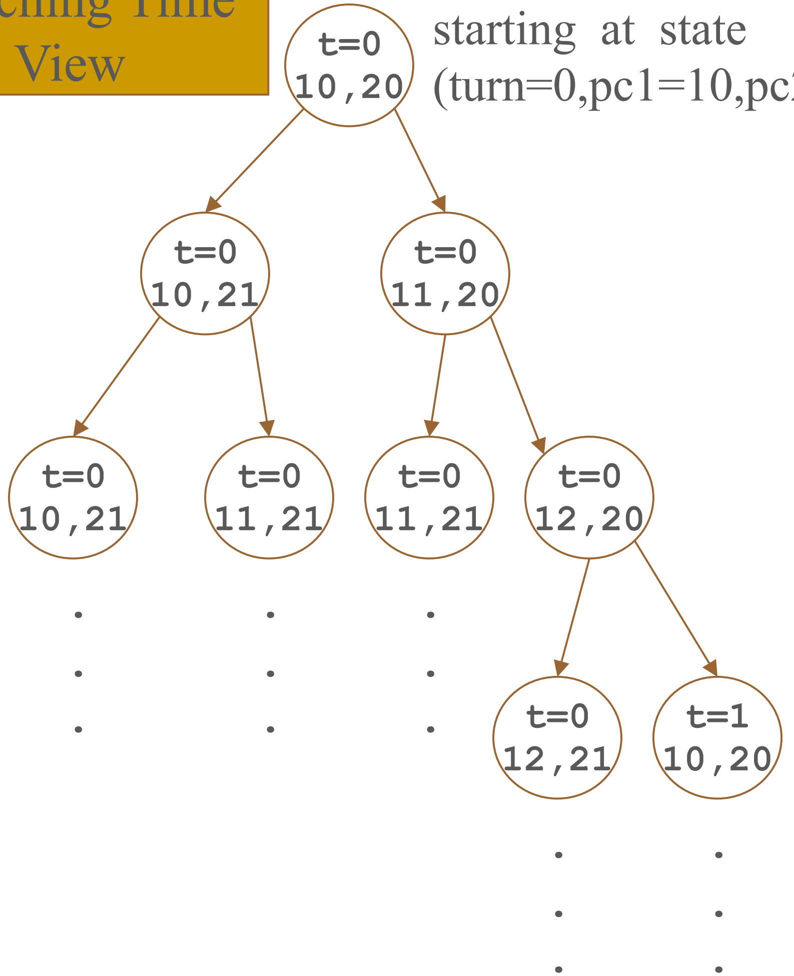
One path starting at state (turn=0,pc1=10,pc2=20)

Linear Time View



Branching Time View

A computation tree starting at state (turn=0,pc1=10,pc2=20)



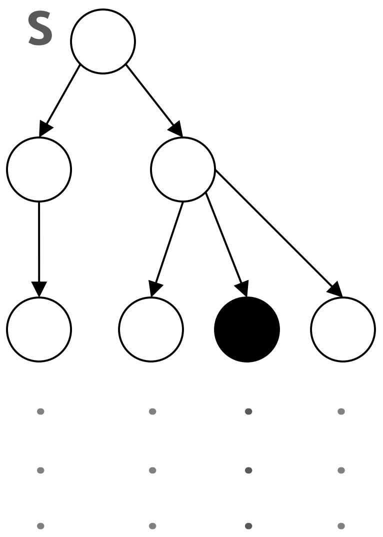
Example/Typical CTL Formulas

- **EF** (*Started* $\wedge$ $\neg$*Ready*): it is possible to get to a state where *Started* holds but *Ready* does not hold.
- **AG** (*Req* $\Rightarrow$ **AF** *Ack*): whenever *Request* occurs, it will be eventually *Acknowledged.*
- **AG** (*DeviceEnabled*): *DeviceEnabled* always holds on every computation path.
- **AG** (**EF** *Restart*): from any state it is possible to get to the *Restart* state.

p does not hold

p holds

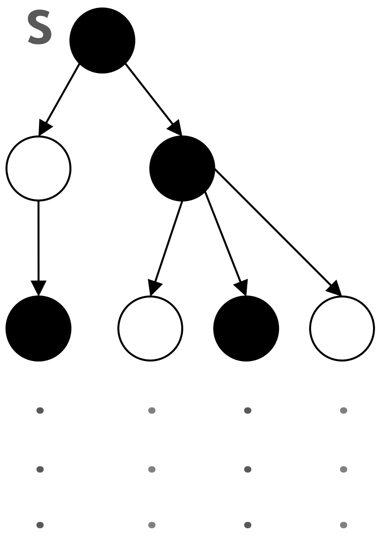
CTL Examples



At state s:

EF p, EX (EX p),
AF (¬p), ¬p holds

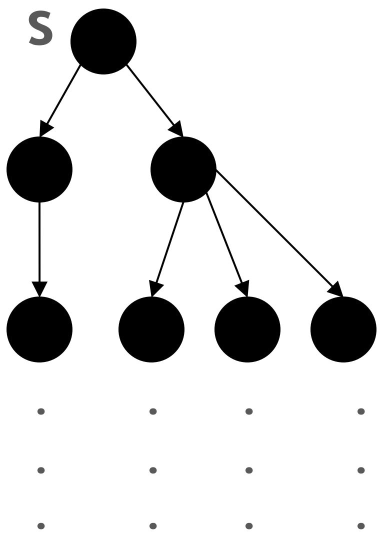
AF p, AG p,
AG (¬p), EX p,
EG p, p does not hold



At state s:

EF p, AF p,
EX (EX p),
EX p, EG p, p holds

AG p, AG (¬p),
AF (¬p) does not hold



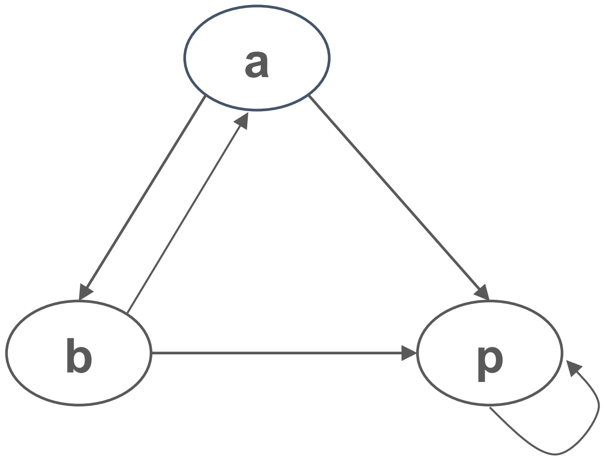
At state s:

EF p, AF p,
AG p, EG p,
Ex p, AX p, p holds

EG (¬ p), EF (¬p),
does not hold

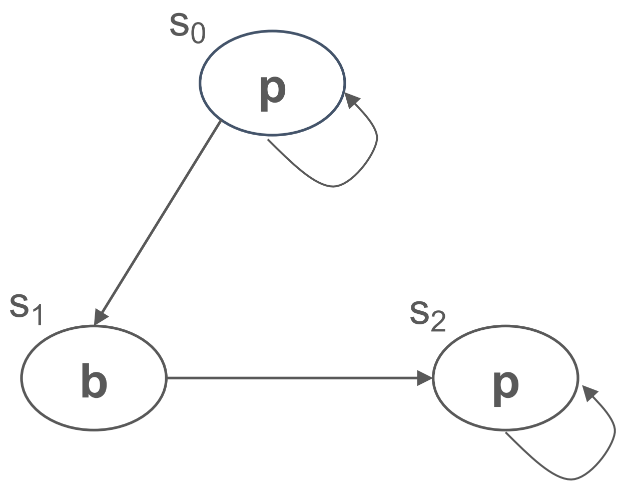
Trivia

- **AG**(**EF** *p*) cannot be expressed in LTL
  - Reset property: from every state it is possible to get to *p*
    - But there might be paths where you never get to *p*
  - Different from **A**(**GF** *p*)
    - Along each possible path, for each state in the path, there is a future state where *p* holds
    - Counterexample: ababab…



Trivia

- A(FG p) cannot be expressed in CTL
  - Along all paths, one eventually reaches a point where p always holds from then on
    - But at some points in some paths where p always holds, there might be a diverging path where p does not hold
  - Different from AF(AG p)
    - Along each possible path there exists a state such that p always holds from then on
    - Counterexample: the path that stays in s0



Linear vs Branching-Time logics

- LTL is a linear time logic: when determining if a path satisfies an LTL formula we are only concerned with **a single path**
- CTL is a branching time logic: when determining if a state satisfies a CTL formula we are concerned **with multiple paths**
  - The computation is viewed as a tree which contains all the paths
  - The computation tree is obtained by unrolling the transition relation
- The expressive powers of CTL and LTL are incomparable (LTL $\subseteq$ CTL*, CTL $\subseteq$ CTL*)
  - Basic temporal properties can be expressed in both logics
  - Not in this lecture, sorry! (Take a class on Modal Logics)

Linear vs Branching-Time logics

Some advantages of LTL

- LTL properties are preserved under "abstraction": i.e., if $\mathcal{M}$ "approximates" a more complex model $\mathcal{M}'$, by introducing more paths, then
  - $\mathcal{M} \models \psi \Rightarrow \mathcal{M}' \models \psi$
- "counterexamples" for LTL are simpler: single executions (not trees).
- The automata-theoretic approach to LTL model checking is simpler (no tree automata).
- most properties people are interested in are (anecdotally) linear-time.

Some advantages of BT

- BT allows expression of some useful properties like 'reset'.
- CTL, a limited fragment of the more complete BT logic CTL*, can be model checked in time linear in the formula size (as well as in the transition system).
  - But formulas are usually smaller than models, so this isn't as important as it may first seem.
- Some BT logics, like μ-calculus and CTL, are well-suited for the kind of fixed-point computation scheme used in symbolic model checking.

Software Model Checking?

- Use a finite state programming language, like executable design specifications (Statecharts, xUML, etc.).
- Extract finite state machines from programs written in conventional programming languages
- Unroll the state machine obtained from the executable of the program.
- Use a combination of the state space reduction techniques to avoid generating too many states.
  - **Verisoft (Bell Labs)**
  - **FormalCheck/xUML (UT Austin, Bell Labs)**
  - **ComFoRT (CMU/SEI)**
- *Use static analysis to extract a finite state skeleton from a program, model check the result.*
  - **Bandera** – Kansas State
  - **Java PathFinder** – NASA Ames
  - **SLAM/Bebop** - Microsoft