Program Synthesis, Part 1

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With slide inspiration gratitude to Emina Torlak and Ras Bodik

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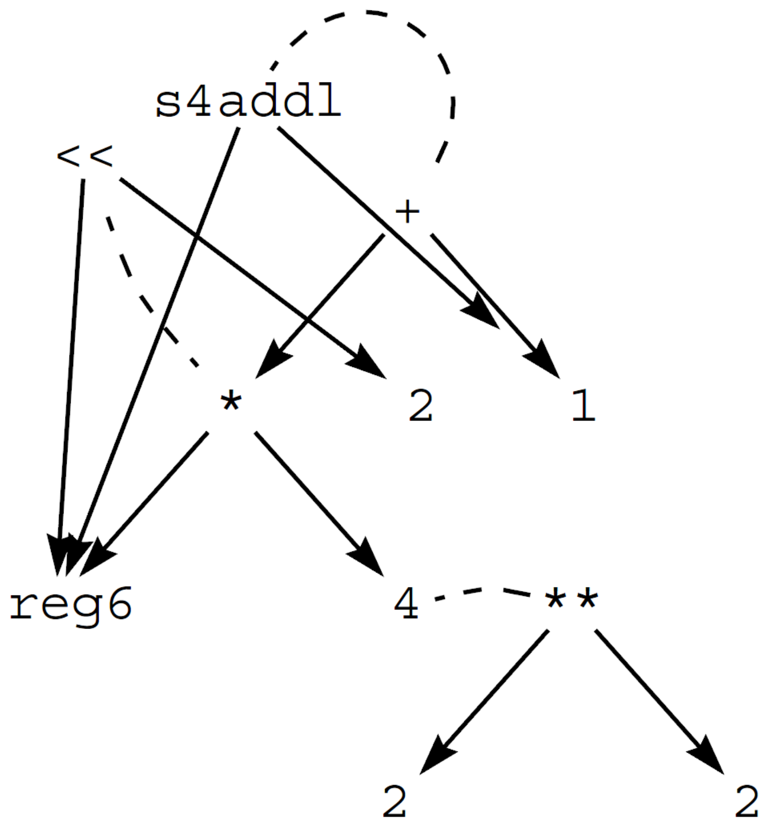
Denali: synthesis with axioms and E-graphs

[Joshi, Nelson, Randall PLDI'02]



$$\forall\, n\,.\; 2^n = 2{*}{*}n$$

$$\forall\, k, n\,.\; k * 2^n = k{<}{<}n$$

$$\forall k, n{:}{:}\; k * 4 + n \;=\; \texttt{s4addl}(k, n)$$

$\texttt{reg6} * 4 + 1 \longrightarrow \texttt{s4addl}(\texttt{reg6}, 1)$

specification — synthesized program

Two kinds of axioms

Instruction semantics: defines (an interpreter for) the language

$$\forall\, k, n\ .\ k * 2^n = k\texttt{<<}n$$

$$\forall k, n{:}:\ k * 4 + n\ = \texttt{s4addl}(k, n)$$

Algebraic properties: associativity of add64, memory modeling, …

$$\forall\, n\ .\ 2^n = 2\texttt{**}n$$

$$(\forall\, x, y :: \texttt{add64}(x, y) = \texttt{add64}(y, x))$$

$$(\forall\, x, y, z :: \texttt{add64}(x, \texttt{add64}(y, z)) = \texttt{add64}(\texttt{add64}(x, y), z))$$

$$(\forall\, x :: \texttt{add64}(x, 0) = x)$$

$$(\forall\, a, i, j, x\ :: i = j \\ \lor \texttt{select}(\texttt{store}(a, i, x), j) = \texttt{select}(a, j))$$

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Compilation vs. synthesis

So where's the line between compilation & synthesis?

Compilation:

1) represent source program as abstract syntax tree (AST)
   - (i) parsing, (ii) name analysis, (iii) type checking
2) lower the AST from source to target language
   - eg, assign machine registers to variables, select instructions, …

Lowering performed with <u>tree rewrite rules,</u> sometimes based on <u>analysis of the program</u>

eg, a variable cannot be in a register if its address is in another variable

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Properties of deductive synthesizers

Efficient and provably correct

- thanks to semantics-preserving rules
- only correct programs are explored
- Denali is scalable: prior super-optimizers gen-and-test

Successfully built for axiomatizable domains

- expression equivalence (Denali)
- linear filters (FFTW, Spiral)
- linear algebra (FLAME)
- statistical calculations (AutoBayes)
- data structures as relational DBs (P2; Hawkins et al.)

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Inductive synthesis

Find a program correct on a set of inputs and hope (or verify) that it's correct on rest of inputs.

- A **partial program** syntactically defines the candidate space.
- Inductive synthesis search phrased as a **constraint problem**.

Program found by (symbolic) interpretation of a (space of) candidates, not by deriving the candidate.

- So, to find a program, we need only an interpreter, not a sufficient set of derivation axioms.

Partial or multi-modal specification of the desired program

Solves $\exists$ P.$\varphi(x_1, P(x_1)) \wedge \ldots \wedge \varphi(x_n\ P(x_n))$ for representative inputs $x_1,\ldots,x_n$

A program P from the given space of candidates that satisfies φ on all (usually bounded) inputs

reg6 * 4 + 1

CEGIS: Counterexample-guided Inductive Synthesis [Solar-Lezama et al., ASPLOS 06]

s4addl(reg6, 1)

expr :=
  const | reg6 |
  s4addl(*expr*, *expr*) |
  ...

A syntactic *sketch* describing the shape of the desired program; defines the space of candidate programs to search. Can be tuned for performance.

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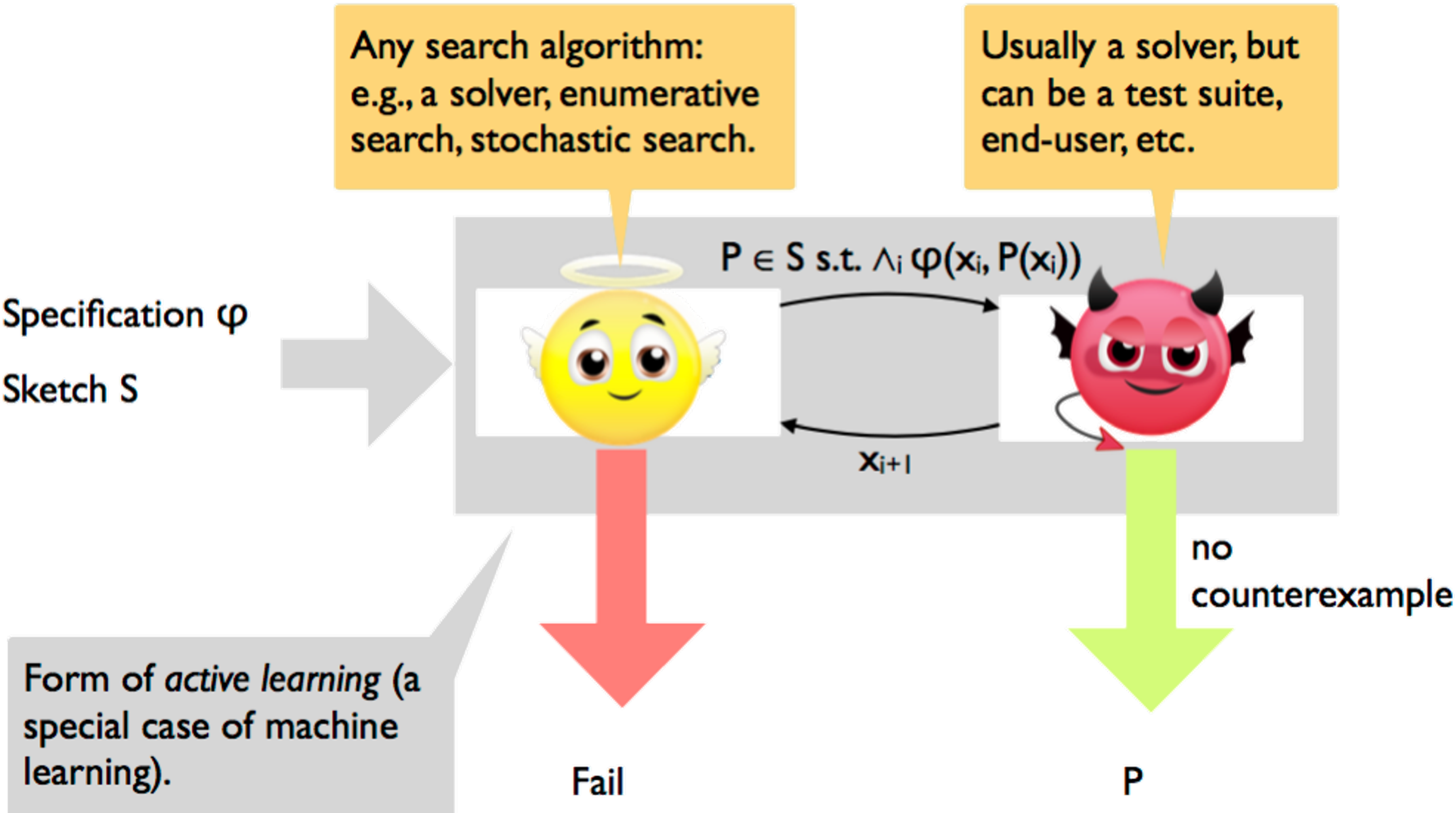
Overview of CEGIS



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Sketching intuition

Extend the language with two constructs

spec:

```
int foo (int x) {
  return x + x;
}
```

$\phi(x, y): y = \textbf{foo}(x)$

sketch:

```
int bar (int x) implements foo {
  return x << ??;
}
```

?? substituted with an int constant meeting ϕ

result:

```
int bar (int x) implements foo {
  return x << 1;
}
```

EXAMPLE: POPULATION COUNTING

```
bit[W] pop (bit[W] x) pop
{
   int count = 0;
   for (int i = 0; i < W; i++) {
     if (x[i]) count++;
   }
   return count;
}
```

```
bit[W] popSketched (bit[W] x)
        implements pop {
    loop (??) {
        x = (x & ??) +
                      ((x >> ??) & ?? );
    }
  return x;
}
```

```
bit[W] popSketched (bit[W] x)
{
  x = (x & 0x5555) +
              ((x >> 1) & 0x5555);
  x = (x & 0x3333) +
              ((x >> 2) & 0x3333);
  x = (x & 0x0077) +
              ((x >> 8) & 0x0077);
  x = (x & 0x000F) +
              ((x >> 4) & 0x000F);
  return x;
}
```

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High level steps

- Write a program *sketch* with holes and a specification.
- A partial evaluator iteratively rewrites program, converts to a Quantified Boolean Formula Satisfiability problem (QBF); problem becomes:

  $\exists\, c \in \{0, 1\}^k, \forall\, x \in \{0, 1\}^m; P(x) = S(x,C)$

  (actually 2QBF, which makes it tractable)
- Use cooperating theorem provers to fill in holes.

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Simple example

```
def f(int[4] in) {
    loop(??)
        f = f ^ in[??] ;
}
```

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Partially evaluated

```
def f(int[4] in, int c1, int c2, int c3, int c4)
{
  let t0 = c1 in
    if(t0>0)
      f = f^in[c2];
      let t1 = t0-1 in
      if(t1>0)
        f = f^in[c3];
        let t2 = t1-1 in
        if(t2>0)
          f = f ^ in[c4];
          assert t2-1 == 0;
}
```

```
function synthesize (sketch S, spec P)
// synthesize control that completes S for a random input;
// check if it works for all other inputs
// if not, add counter example input to set of inputs and repeat
I = {}
x = random()
do
  I = I ∪ { x }
  c = synthesizeForSomeInputs(I)
  if c = nil then exit (“buggy sketch”)
  x = verifyForAllInputs(c)
while x ≠ nil
return c

function synthesizeForSomeInputs(inputs set I)
// synthesize controls C that make the sketch equivalent to the
// specification on all inputs from I
if ∧_{x ∈ I} P(x) = S(x,c) is satisfiable then
  return a satisfying c
else
  return nil

function verifyForAllInputs(control c)
// verify if sketch S completed with controls c is functionally
// equivalent to the specification P.  If not, return the counterexample
if P(x) ≠ S(x, c) is satisfiable then
  return satisfying x
else
  return nil
```


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Example: Parallel Matrix Transpose

Example: 4x4-matrix transpose with SIMD

a functional (executable) specification:

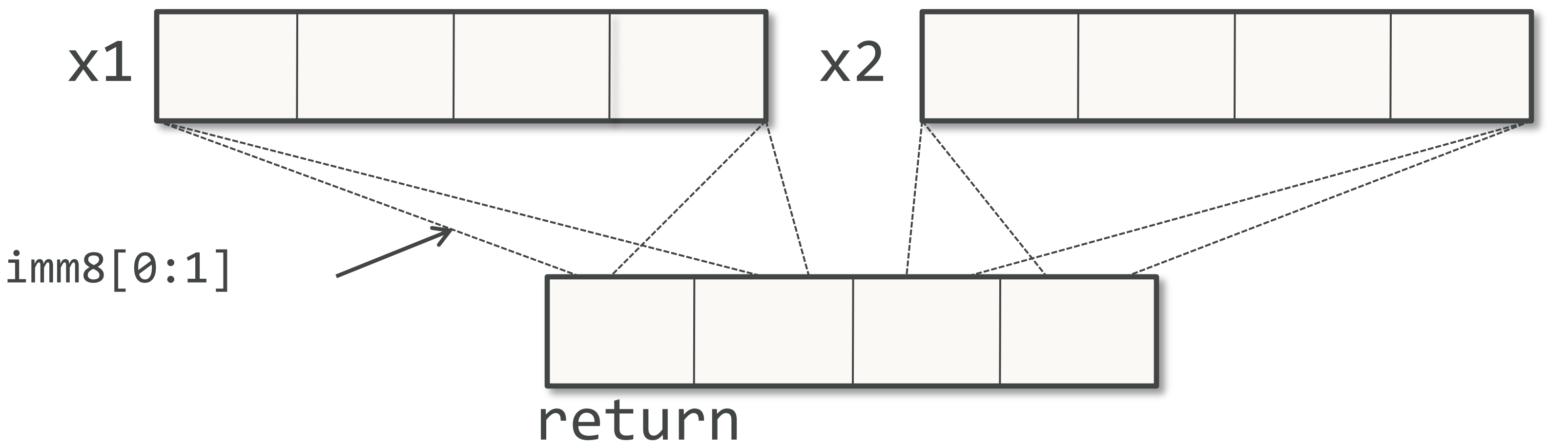
```
int[16] transpose(int[16] M) {
  int[16] T = 0;
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      T[4 * i + j] = M[4 * j + i];
  return T;
}
```

(This example comes from a Sketch grad-student contest.)

Implementation idea: parallelize with SIMD

Intel SHUFP (shuffle parallel scalars) SIMD instruction:

```
return = shufps(x1, x2, imm8 :: bitvector8)
```



High-level insight: transpose as a 2-phase shuffle

- Matrix M can be transposed in two shuffle phases
  - Phase 1: shuffle M into an intermediate matrix S with some number of shufps instructions
  - Phase 2: shuffle S into the result matrix T with some number of shufps instructions
- Synthesis with partial programs helps one to complete their insight. Or prove it wrong.

SIMD matrix transpose, sketched

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;

  S[??::4] = shufps(M[??::4], M[??::4], ??);
  S[??::4] = shufps(M[??::4], M[??::4], ??);
  …
  S[??::4] = shufps(M[??::4], M[??::4], ??);

  T[??::4] = shufps(S[??::4], S[??::4], ??);
  T[??::4] = shufps(S[??::4], S[??::4], ??);
  …
  T[??::4] = shufps(S[??::4], S[??::4], ??);

  return T;
}
```

Phase 1

Phase 2

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SIMD matrix transpose with more insight

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;

  S[??::4] = shufps(M[??::4], M[??::4], ??);
  S[??::4] = shufps(M[??::4], M[??::4], ??);
  S[??::4] = shufps(M[??::4], M[??::4], ??);
  S[??::4] = shufps(M[??::4], M[??::4], ??);

  T[??::4] = shufps(S[??::4], S[??::4], ??);
  T[??::4] = shufps(S[??::4], S[??::4], ??);
  T[??::4] = shufps(S[??::4], S[??::4], ??);
  T[??::4] = shufps(S[??::4], S[??::4], ??);

  return T;
}
```

4 shuffle instructions per phase

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SIMD matrix transpose with even more insight

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;

  S[0::4]  = shufps(M[??::4], M[??::4], ??);
  S[4::4]  = shufps(M[??::4], M[??::4], ??);
  S[8::4]  = shufps(M[??::4], M[??::4], ??);
  S[12::4] = shufps(M[??::4], M[??::4], ??);

  T[0::4]  = shufps(S[??::4], S[??::4], ??);
  T[4::4]  = shufps(S[??::4], S[??::4], ??);
  T[8::4]  = shufps(S[??::4], S[??::4], ??);
  T[12::4] = shufps(S[??::4], S[??::4], ??);

  return T;
}
```

1 shuffle instruction per row of output

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SIMD matrix transpose, sketched

```
int[16] trans_sse(int[16] M) implements trans {
  int[16] S = 0, T = 0;
  repeat (??) S[??::4] = shufps(M[??::4], M[??::4], ??);
  repeat (??) T[??::4] = shufps(S[??::4], S[??::4], ??);
  return T;
}
```

From the contestant email:
`Over the summer, I spent about 1/2 a day manually figuring it out.`
`Synthesis time:` `< 2 minutes.`

```
int[16] trans_sse(int[16] M) implements trans { // synthesized code
  S[4::4]   = shufps(M[6::4],   M[2::4],  11001000b);
  S[0::4]   = shufps(M[11::4],  M[6::4],  10010110b);
  S[12::4]  = shufps(M[0::4],   M[2::4],  10001101b);
  S[8::4]   = shufps(M[8::4],   M[12::4], 11010111b);
  T[4::4]   = shufps(S[11::4],  S[1::4],  10111100b);
  T[12::4]  = shufps(S[3::4],   S[8::4],  11000011b);
  T[8::4]   = shufps(S[4::4],   S[9::4],  11100010b);
  T[0::4]   = shufps(S[12::4],  S[0::4],  10110100b);
}
```